On the vacuum of the minimal nonsupersymmetric $SO(10)$ unification

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We study a class of nonsupersymmetric $SO(10)$ grand unified scenarios where the first stage of the symmetry breaking is driven by the vacuum expectation values of the 45-dimensional adjoint representation. Three decade old results claim that such a Higgs setting may lead exclusively to the flipped $SU(5) \otimes U(1)$ intermediate stage. We show that this conclusion is actually an artifact of the tree level potential. The study of the accidental global symmetries emerging in various limits of the scalar potential offers a simple understanding of the tree level result and a rationale for the drastic impact of quantum corrections. We scrutinize in detail the simplest and paradigmatic case of the $45_H \oplus 16_H$ Higgs sector triggering the breaking of $SO(10)$ to the standard electroweak model. We show that the minimization of the one-loop effective potential allows for intermediate $SU(4)_C \otimes SU(2)_L \otimes U(1)_R$ and $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ symmetric stages as well. These are the options favored by gauge unification. Our results, that apply whenever the $SO(10)$ breaking is triggered by $\langle 45_H \rangle$, open the path for hunting the simplest realistic scenario of nonsupersymmetric $SO(10)$ grand unification.

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I. INTRODUCTION

The Grand Unified Theories (GUTs) [1] qualify among the most appealing physics scenarios beyond the Standard Model (SM) of electroweak and strong interactions. Though being under scrutiny for about 35 years they still attract a lot of attention across the high energy community due to their intrinsic predictivity and to their potential for understanding the origin of our low energy world texture. Apart from offering definite experimental motivations for e.g. proton decay or monopole searches, GUTs typically give rise to non-trivial correlations among observables associated to different SM sectors. The most prominent of these is the consistent determination of the weak-mixing angle and the strong coupling arising from the gauge coupling unification in a weak scale supersymmetric scenario.

In recent years, an extra boost to the field was triggered by the discovery of non-zero neutrino masses in the sub-eV region. Within the grand-unified scenarios this discovery translates into constraints on the intermediate scales (typically well separated from the unification scale $M_G \sim 10^{16}$ GeV) underpinning some variant of the seesaw mechanism [2, 3]. Furthermore, the observed peculiarity of the lepton mixing pattern [4] challenges the flavour structure of the simplest models due to the strong correlations in the Yukawa sector. In this respect, the requirement of minimality, that stands for the simplicity of the relevant Higgs sector, is a valuable guiding principle for model building.

On this basis, it has been argued recently that the minimal supersymmetric $SO(10)$ model [5–7] is indeed incompatible with the electroweak flavour constraints [8]. The minimal supersymmetric setting suffers from an inherent proximity of the GUT and the seesaw scales, at odds with the lower bound on the neutrino mass scale implied by the oscillation phenomena. The proposed ways out (resorting e.g. to a non-minimal Higgs sector [9] or invoking split supersymmetry [10]) hardly pair the appeal of the minimal setting.

Were a large (GUT scale) breaking of global supersymmetry be at play (a possible LHC test of this hypothesis has been recently put forward in Ref. [11]), then baryon number violating $d = 5$ operators decouple from our low-energy world and gauge unification exhibits naturally the required splitting between the seesaw and the GUT scales [12–15]. Nevertheless, devising a realistic and simple enough $SO(10)$ GUT along these lines remains a rather non-trivial task.

The main reason has to do with the structure of the minimal Higgs sector of nonsupersymmetric $SO(10)$ models. A full breaking of the GUT symmetry down to the SM can be achieved via a pair of Higgs multiplets[1]: one 45-dimensional adjoint representation, 45_H,

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[1] The authors in Ref. [16] observe that a one-step breaking of $SO(10)$ can be achieved via one 144 ($\overline{144}$) Higgs representation. Such a setting, suitable for a supersymmetric gauge unification, requires an extended matter sector, including 45 and 120 multiplets, in order to accommodate realistic fermion masses [17].

and one 16-dimensional spinorial representation, 16_H (or one 126-dimensional tensor representation 126_H). A SM preserving breaking pattern is controlled by two 45_H vacuum expectation values (VEVs) and one 16_H (or 126_H) VEV. Different configurations of the two adjoint VEVs preserve different $SO(10)$ subalgebras, namely, $4_C\,2_L\,1_R$, (short-hand notation for $SU(4)_c \otimes SU(2)_L \otimes U(1)_R$), $3_c\,2_L\,2_R\,1_{B-L}$, $3_c\,2_L\,1_R\,1_{B-L}$, and the flipped or standard $SU(5) \otimes U(1)$. Except for the latter case, the subsequent breaking to the SM is obtained via the standard $SU(5)$ conserving 16_H (or 126_H) VEV.

Remarkably enough, a consistent $SO(10)$ gauge symmetry breaking in the usual low-scale supersymmetric context requires minimally $45_H \oplus 54_H$ [18] (or 210_H [6] in the renormalizable variant), in addition to $\overline{16}_H \oplus 16_H$ (or $\overline{126}_H \oplus 126_H$).

The phenomenologically favored scenarios allowed by gauge coupling unification correspond minimally to a two-step breaking along one of the following directions [15]:

$$SO(10) \xrightarrow{M_G} 3_c\,2_L\,2_R\,1_{B-L} \xrightarrow{M_I} \text{SM}\,, \qquad (1)$$

$$SO(10) \xrightarrow{M_G} 4_C\,2_L\,1_R \xrightarrow{M_I} \text{SM}\,, \qquad (2)$$

where the first breaking stage is driven by the 45_H VEVs, while the breaking to the SM at the intermediate scale M_I, several orders of magnitude below the unification scale M_G, is controlled by the 16_H (or 126_H) VEV. One of the two 45_H VEVs may also contribute to the second step (see the discussion on the required intermediate scale Higgs multiplets in Ref. [15] and in Sect. V F).

Gauge unification, even without proton decay limits, excludes any intermediate $SU(5)$-symmetric stages. On the other hand, a series of studies in the early 1980's of the $45_H \oplus 16_H$ model [19–21] indicated that the only intermediate stages allowed by the scalar sector dynamics were the flipped $SU(5) \otimes U(1)$ for leading 45_H VEVs or the standard $SU(5)$ GUT for dominant 16_H VEV.

This observation excluded the simplest $SO(10)$ Higgs sector from realistic consideration.

In this paper we show that the exclusion of the breaking patterns in Eqs. (1)–(2) is an artifact of the tree level potential. As a matter of fact, some entries of the scalar hessian are accidentally over-constrained at the tree level. A number of scalar interactions that, by a simple inspection of the relevant global symmetries and their explicit breaking, are expected to contribute to these critical entries, are not effective at the tree level.

On the other hand, once quantum corrections are considered, contributions of $O(M_G^2/16\pi^2)$ induced on these entries open in a natural way all group-theoretically allowed vacuum configurations. Remarkably enough, the study of the one-loop effective potential can be consistently carried out just for the critical tree level hessian entries (that correspond to specific pseudo-Goldstone boson masses). For all other states in the scalar spectrum, quantum corrections remain perturbations of the tree

level results and do not affect the discussion of the vacuum pattern.

Our conclusions apply to any Higgs setting where the first step of the $SO(10)$ gauge symmetry breaking is driven by the 45_H VEVs, while the other Higgs representations control the intermediate and weak scale stages. The results presented here and in Ref. [15] do open the path towards a realistic nonsupersymmetric $SO(10)$ unification. A detailed study of minimal setups will be the subject of a future work.

The paper is organized as follows. The study of the tree-level scalar potential and the related scalar mass spectrum are concisely reviewed in Sects. II–III. A detailed understanding of the mass textures is developed in Sect. IV in terms of a systematic discussion of the accidental global symmetries and the associated pseudo-Goldstone bosons. In Sect. V we calculate the relevant quantum corrections by means of the one-loop effective potential, and we prove the existence of the new vacua. The main results and the prospects for further developments are summarized in Sect. VI. Most of the technical aspects of the work are deferred to Apps. A–D.

II. THE MINIMAL SO(10) GUT

In this study we consider a nonsupersymmetric $SO(10)$ setup featuring the minimal Higgs content sufficient to trigger the spontaneous breakdown of the GUT symmetry down to the standard electroweak model. Minimally, the scalar sector spans over a reducible $45_H \oplus 16_H \oplus 10_H$ representation. The adjoint 45_H and the spinor 16_H multiplets contain three SM singlets that may acquire GUT scale VEVs.

The 10_H, which together with the relevant components of 16_H triggers the SM $\to SU(3)_c \otimes U(1)_Q$ breaking, is introduced in order to admit for a potentially realistic fermionic spectrum. The VEV of 10_H is very tiny in comparison with the VEVs of 45_H or 16_H and in the chains we are considering it mixes with 16_H only at the electroweak scale. On the other hand, most of the 10_H component fields do maintain a natural mass of the order of the unification scale. In this respect they play a role also for the details of the theory at the GUT scale. Nevertheless, as we shall see, the 10_H is not needed for the scope of the present discussion and we shall neglect it altogether.

Let us emphasize once more that the issue we shall be dealing with is inherent to all nonsupersymmetric $SO(10)$ models with one adjoint 45_H governing the first breaking step. Only one additional scalar representation interacting with the adjoint is sufficient to demonstrate conclusively our claim. In this respect, the choice of the $SO(10)$ spinor to trigger the intermediate symmetry breakdown is a mere convenience and a similar line of reasoning can be devised for the scenarios in which $B-L$ is broken for instance by a 126-dimensional $SO(10)$ tensor.

We shall therefore study the structure of the vacua

of a $SO(10)$ Higgs potential with only the $45_H \oplus 16_H$ representation at play. Following the common convention, we define $16_H \equiv \chi$ and denote by χ_+ and χ_- the multiplets transforming as positive and negative chirality components of the reducible 32-dimensional $SO(10)$ spinor representation respectively Similarly, we shall use the symbol Φ (or the derived ϕ, c.f. Appendix A) for the adjoint Higgs representation 45_H (or its components in the natural basis).

The minimal $SO(10)$ GUT accommodates the SM matter in three copies of $SO(10)$ spinors 16_F^i, $(i = 1, 2, 3)$. The fermions (and their Yukawa interactions) do not play any role in the GUT scale dynamics and will not be considered further (we assume the masses of the right-handed neutrinos to be small with respect to the unification scale). The detailed study of realistic Higgs and Yukawa sectors will be the subject of a forthcoming paper.

A. The tree-level Higgs potential

The most general renormalizable tree-level scalar potential which can be constructed out of 45_H and 16_H reads (see for instance Refs. [22, 23]):

$$V_0 = V_\Phi + V_\chi + V_{\Phi\chi}, \tag{3}$$

where, according to the notation in Appendix A,

$$V_\Phi = -\frac{\mu^2}{2}\mathrm{Tr}\,\Phi^2 + \frac{a_1}{4}(\mathrm{Tr}\,\Phi^2)^2 + \frac{a_2}{4}\mathrm{Tr}\,\Phi^4, \tag{4}$$
$$V_\chi = -\frac{\nu^2}{2}\chi^\dagger\chi + \frac{\lambda_1}{4}(\chi^\dagger\chi)^2 + \frac{\lambda_2}{4}(\chi_+^\dagger\Gamma_j\chi_-)(\chi_-^\dagger\Gamma_j\chi_+)$$

and

$$V_{\Phi\chi} = \alpha(\chi^\dagger\chi)\mathrm{Tr}\,\Phi^2 + \beta\chi^\dagger\Phi^2\chi + \tau\chi^\dagger\Phi\chi. \tag{5}$$

The mass terms and coupling constants above are real by hermiticity. Linear and cubic Φ self-interactions are absent due the zero trace of the $SO(10)$ adjoint representation. For the sake of simplicity, all tensorial indices have been suppressed.

B. The symmetry breaking patterns

1. The SM singlets

There are in general three SM singlets in the $45_H \oplus 16_H$ representation of $SO(10)$. Using $(B - L)/2 \equiv X$ and labeling the field components according to $3_c\,2_L\,2_R\,1_X$, the SM singlets reside in the $(1, 1, 1, 0)$ and $(1, 1, 3, 0)$ submultiplets of 45_H and in the $(1, 1, 2, +\frac{1}{2})$ component of 16_H. We denote their VEVs as

$$\begin{aligned}
\langle(1, 1, 1, 0)\rangle &\equiv \omega_Y, \\
\langle(1, 1, 3, 0)\rangle &\equiv \omega_R, \\
\langle(1, 1, 2, +\tfrac{1}{2})\rangle &\equiv \chi_R,
\end{aligned} \tag{6}$$

where $\omega_{Y,R}$ are real and χ_R can be taken real by a phase redefinition of the 16_H. Different VEV configurations trigger the spontaneous breakdown of the $SO(10)$ symmetry into a number of subgroups. Namely, for $\chi_R = 0$ one finds

$$\begin{aligned}
\omega_R = 0, \, \omega_Y \neq 0 &: & 3_c\,2_L\,2_R\,1_X \\
\omega_R \neq 0, \, \omega_Y = 0 &: & 4_C 2_L 1_R \\
\omega_R \neq 0, \, \omega_Y \neq 0 &: & 3_c\,2_L\,1_R\,1_X \\
\omega_R = -\omega_Y \neq 0 &: & \text{flipped } 5'\,1_{Z'} \\
\omega_R = \omega_Y \neq 0 &: & \text{standard } 5\,1_Z
\end{aligned} \tag{7}$$

with $5\,1_Z$ and $5'\,1_{Z'}$ standing for the two different embedding of the $SU(5)$ subgroup into $SO(10)$, i.e. standard and "flipped" respectively (see the discussion at the end of the section).

When $\chi_R \neq 0$ all intermediate gauge symmetries are spontaneously broken down to the SM group, with the exception of the last case which maintains the standard $SU(5)$ subgroup unbroken and will no further be considered.

The classification in Eq. (7) depends on the phase conventions used in the parametrization of the SM singlet subspace of $45_H \oplus 16_H$. The statement that $\omega_R = \omega_Y$ yields the standard $SU(5)$ vacuum while $\omega_R = -\omega_Y$ corresponds to the flipped setting defines a particular basis in this subspace (see Sect. II B 3). The consistency of any chosen framework is then verified against the corresponding Goldstone boson spectrum.

The decomposition of the 45_H and 16_H representations with respect to the relevant $SO(10)$ subgroups is detailed in Tables I and II.

2. The L-R chains

According to the analysis in Ref. [15], the potentially viable breaking chains fulfilling the basic gauge unification constraints (with a minimal $SO(10)$ Higgs sector) correspond to the settings:

$$\begin{aligned}
\omega_Y \gg \omega_R > \chi_R &: \\
SO(10) \to 3_c 2_L 2_R 1_X &\to 3_c 2_L 1_R 1_X \to 3_c 2_L 1_Y
\end{aligned} \tag{8}$$

$$\begin{aligned}
\omega_R \gg \omega_Y > \chi_R &: \\
SO(10) \to 4_C 2_L 1_R &\to 3_c 2_L 1_R 1_X \to 3_c 2_L 1_Y
\end{aligned} \tag{9}$$

As remarked in [15], the cases $\chi_R \sim \omega_R$ or $\chi_R \sim \omega_Y$ lead to effective two-step $SO(10)$ breaking patterns with a non-minimal set of surviving scalars at the intermediate scale. On the other hand, a truly two-step setup can be recovered (with a minimal fine tuning) by considering the cases where ω_R or ω_Y exactly vanish. Only the explicit study of the scalar potential determines which of the textures are allowed.

We have verified that in all cases the GUT thresholds effects related to the relevant pseudo-Goldstone mass

$4_C\,2_L\,2_R$	$4_C\,2_L\,1_R$	$3_c\,2_L\,2_R\,1_X$	$3_c\,2_L\,1_R\,1_X$	$3_c\,2_L\,1_Y$	5	$5'\,1_{Z'}$	$1_{Y'}$
$(4,2,1)$	$(4,2,0)$	$\left(3,2,1,+\frac16\right)$	$\left(3,2,0,+\frac16\right)$	$\left(3,2,+\frac16\right)$	10	$(10,+1)$	$+\frac16$
		$\left(1,2,1,-\frac12\right)$	$\left(1,2,0,-\frac12\right)$	$\left(1,2,-\frac12\right)$	$\overline{5}$	$(\overline{5},-3)$	$-\frac12$
$(\overline{4},1,2)$	$\left(\overline{4},1,+\frac12\right)$	$\left(\overline{3},1,2,-\frac16\right)$	$\left(\overline{3},1,+\frac12,-\frac16\right)$	$\left(\overline{3},1,+\frac13\right)$	$\overline{5}$	$(10,+1)$	$-\frac23$
	$\left(\overline{4},1,-\frac12\right)$		$\left(\overline{3},1,-\frac12,-\frac16\right)$	$\left(\overline{3},1,-\frac23\right)$	10	$(\overline{5},-3)$	$+\frac13$
		$\left(1,1,2,+\frac12\right)$	$\left(1,1,+\frac12,+\frac12\right)$	$(1,1,+1)$	10	$(1,+5)$	0
		$\left(1,1,-\frac12,+\frac12\right)$	$(1,1,0)$		1	$(10,+1)$	$+1$

TABLE I. Decomposition of the spinorial representation 16 with respect to the various $SO(10)$ subgroups. The definitions and normalization of the abelian charges are given in the text.

$4_C\,2_L\,2_R$	$4_C\,2_L\,1_R$	$3_c\,2_L\,2_R\,1_X$	$3_c\,2_L\,1_R\,1_X$	$3_c\,2_L\,1_Y$	5	$5'\,1_{Z'}$	$1_{Y'}$
$(1,1,3)$	$(1,1,+1)$	$(1,1,3,0)$	$(1,1,+1,0)$	$(1,1,+1)$	10	$(10,-4)$	$+1$
	$(1,1,0)$		$(1,1,0,0)$	$(1,1,0)$	1	$(1,0)$	0
	$(1,1,-1)$		$(1,1,-1,0)$	$(1,1,-1)$	$\overline{10}$	$(\overline{10},+4)$	-1
$(1,3,1)$	$(1,3,0)$	$(1,3,1,0)$	$(1,3,0,0)$	$(1,3,0)$	24	$(24,0)$	0
$(6,2,2)$	$\left(6,2,+\frac12\right)$	$\left(3,2,2,-\frac13\right)$	$\left(3,2,+\frac12,-\frac13\right)$	$\left(3,2,\frac16\right)$	10	$(24,0)$	$-\frac56$
	$\left(6,2,-\frac12\right)$		$\left(3,2,-\frac12,-\frac13\right)$	$\left(3,2,-\frac56\right)$	24	$(10,-4)$	$+\frac16$
		$\left(\overline{3},2,2,+\frac13\right)$	$\left(\overline{3},2,+\frac12,+\frac13\right)$	$\left(\overline{3},2,+\frac56\right)$	24	$(\overline{10},+4)$	$-\frac16$
			$\left(\overline{3},2,-\frac12,+\frac13\right)$	$\left(\overline{3},2,-\frac16\right)$	$\overline{10}$	$(24,0)$	$+\frac56$
$(15,1,1)$	$(15,1,0)$	$(1,1,1,0)$	$(1,1,0,0)$	$(1,1,0)$	24	$(24,0)$	0
		$\left(3,1,1,+\frac23\right)$	$\left(3,1,0,+\frac23\right)$	$\left(3,1,+\frac23\right)$	$\overline{10}$	$(\overline{10},+4)$	$+\frac23$
		$\left(\overline{3},1,1,-\frac23\right)$	$\left(\overline{3},1,0,-\frac23\right)$	$\left(\overline{3},1,-\frac23\right)$	10	$(10,-4)$	$-\frac23$
		$(8,1,1,0)$	$(8,1,0,0)$	$(8,1,0)$	24	$(24,0)$	0

TABLE II. Same as in Table I for the $SO(10)$ adjoint (45) representation.

patterns obtained in the present analysis fully comply with the unification constraints in Ref. [15]. Furthermore, the lower bounds on the position of the $B-L$ scale are consistently increased, hence improving the prospects for a successful model building.

3. Standard SU(5) versus flipped SU(5)

There are in general two distinct SM-compatible embeddings of $SU(5)$ into $SO(10)$ [24, 25]. They differ in one generator of the $SU(5)$ Cartan algebra and therefore in the $U(1)_Z$ cofactor.

In the "standard" embedding, the weak hypercharge operator $Y = T_R^{(3)} + T_X$ belongs to the $SU(5)$ algebra and the orthogonal Cartan generator Z (obeying $[T_i, Z] = 0$ for all $T_i \in SU(5)$) is given by $Z = -4T_R^{(3)} + 6T_X$.

In the "flipped" $SU(5)'$ case, the right-handed isospin assignment of quark and leptons into the $SU(5)'$ multiplets is turned over so that the "flipped" hypercharge generator reads $Y' = -T_R^{(3)} + T_X$. Accordingly, the additional $U(1)_{Z'}$ generator reads $Z' = 4T_R^{(3)} + 6T_X$, such that $[T_i, Z'] = 0$ for all $T_i \in SU(5)'$. Weak hypercharge is then given by $Y = (Z' - Y')/5$.

Tables I–II show the standard and flipped $SU(5)$ de-

compositions of the spinorial and adjoint $SO(10)$ representations respectively.

The two $SU(5)$ vacua in Eq. (7) differ by the texture of the adjoint representation VEVs: in the standard $SU(5)$ case they are aligned with the Z operator while they match the Z' structure in the flipped $SU(5)'$ setting (see Appendix A 4 for an explicit representation).

III. THE CLASSICAL VACUUM

A. The stationarity conditions

By substituting Eq. (6) into Eq. (3) the vacuum manifold reads

$$
\begin{aligned}
\langle V_0 \rangle =\ & -2\mu^2(2\omega_R^2 + 3\omega_Y^2) + 4a_1(2\omega_R^2 + 3\omega_Y^2)^2 \\
& + \frac{a_2}{4}(8\omega_R^4 + 21\omega_Y^4 + 36\omega_R^2\omega_Y^2) \\
& - \frac{\nu^2}{2}\chi_R^2 + \frac{\lambda_1}{4}\chi_R^4 \\
& + 4\alpha\chi_R^2(2\omega_R^2 + 3\omega_Y^2) + \frac{\beta}{4}\chi_R^2(2\omega_R + 3\omega_Y)^2 \\
& - \frac{\tau}{2}\chi_R^2(2\omega_R + 3\omega_Y)
\end{aligned}
\tag{10}
$$

The corresponding three stationary conditions can be conveniently written as

$$\frac{1}{8}\left(\frac{\partial \langle V_0 \rangle}{\partial \omega_R} - \frac{2}{3}\frac{\partial \langle V_0 \rangle}{\partial \omega_Y}\right) = \left[-\mu^2 + 4a_1(2\omega_R^2 + 3\omega_Y^2) + \frac{a_2}{4}(4\omega_R^2 + 7\omega_Y^2 - 2\omega_Y\omega_R) + 2\alpha\chi_R^2\right](\omega_R - \omega_Y) = 0 \quad (11)$$

$$\omega_Y\frac{\partial \langle V_0 \rangle}{\partial \omega_R} - \omega_R\frac{2}{3}\frac{\partial \langle V_0 \rangle}{\partial \omega_Y} = -\left[4a_2(\omega_R + \omega_Y)\omega_R\omega_Y + \beta\chi_R^2(2\omega_R + 3\omega_Y) - \tau\chi_R^2\right](\omega_R - \omega_Y) = 0 \quad (12)$$

$$\frac{\partial \langle V_0 \rangle}{\partial \chi_R} = \left[-\nu^2 + \lambda_1\chi_R^2 + 8\alpha(2\omega_R^2 + 3\omega_Y^2) + \frac{\beta}{2}(2\omega_R + 3\omega_Y)^2 - \tau(2\omega_R + 3\omega_Y)\right]\chi_R = 0 \quad (13)$$

We have chosen linear combinations that factor out the uninteresting standard $SU(5) \otimes U(1)_Z$ solution, namely $\omega_R = \omega_Y$.

In summary, when $\chi_R = 0$, Eqs. (11)–(12) allow for four possible vacua:

- $\omega = \omega_R = \omega_Y$ (standard $5\,1_Z$)

- $\omega = \omega_R = -\omega_Y$ (flipped $5'\,1_{Z'}$)

- $\omega_R = 0$ and $\omega_Y \neq 0$ ($3_c\,2_L\,2_R\,1_X$)

- $\omega_R \neq 0$ and $\omega_Y = 0$ ($4_C\,2_L\,1_R$)

As we shall see, the last two options are not *tree level* minima. Let us remark that for $\chi_R \neq 0$, Eq. (12) implies naturally a correlation among the 45_H and 16_H VEVs, or a fine tuned relation between β and τ, depending on the stationary solution. In the cases $\omega_R = -\omega_Y$, $\omega_R = 0$ and $\omega_Y = 0$ one obtains $\tau = \beta\omega$, $\tau = 3\beta\omega_Y$ and $\tau = 2\beta\omega_R$ respectively. Consistency with the scalar mass spectrum must be verified in each case.

B. The tree-level spectrum

The gauge and scalar spectra corresponding to the SM vacuum configuration (with non-vanishing VEVs in $45_H \oplus 16_H$) are detailed in Appendix C.

The scalar spectra obtained in various limits of the tree-level Higgs potential, corresponding to the appearance of accidental global symmetries, are derived in Apps. C 2 a–C 2 e. The emblematic case $\chi_R = 0$ is scrutinized in Appendix C 2 f.

C. Constraints on the potential parameters

The parameters (couplings and VEVs) of the scalar potential are constrained by the requirements of boundedness and the absence of tachyonic states, ensuring that the vacuum is stable and the stationary points correspond to physical minima.

Necessary conditions for vacuum stability are derived in Appendix B. In particular, on the $\chi_R = 0$ section one obtains

$$a_1 > -\frac{13}{80}a_2\,. \quad (14)$$

Considering the general case, the absence of tachyons in the scalar spectrum yields among else

$$a_2 < 0\,, \qquad -2 < \omega_Y/\omega_R < -\frac{1}{2}\,. \quad (15)$$

The strict constraint on ω_Y/ω_R is a consequence of the tightly correlated form of the tree-level masses of the $(8, 1, 0)$ and $(1, 3, 0)$ submultiplets of 45_H, labeled according to the SM $(3_c\,2_L\,1_Y)$ quantum numbers, namely

$$M^2(1, 3, 0) = 2a_2(\omega_Y - \omega_R)(\omega_Y + 2\omega_R)\,, \quad (16)$$

$$M^2(8, 1, 0) = 2a_2(\omega_R - \omega_Y)(\omega_R + 2\omega_Y)\,, \quad (17)$$

that are simultaneously positive only if Eq. (15) is enforced. For comparison with previous studies, let us remark that in the $\tau = 0$ limit (corresponding to an extra Z_2 symmetry $\Phi \to -\Phi$) the intersection of the constraints from Eq. (12), Eqs. (16)–(17) and the mass eigenvalues of the $(1, 1, 1)$ and $(3, 2, 1/6)$ states, yields

$$a_2 < 0\,, \quad -1 \leq \omega_Y/\omega_R \leq -\frac{2}{3}\,, \quad (18)$$

thus recovering the results of Refs. [19–21].

In either case, one concludes by inspecting the scalar mass spectrum that flipped $SU(5)' \otimes U(1)_{Z'}$ is for $\chi_R = 0$ the only solution admitted by Eq. (12) consistent with the constraints in Eq. (15) (or Eq. (18)). For $\chi_R \neq 0$, the fine tuned possibility of having or $\omega_Y/\omega_R \sim -1$ such that χ_R is obtained at an intermediate scale fails to reproduce the SM couplings [15]. Analogous and obvious conclusions hold for $\omega_Y \sim \omega_R \sim \chi_R \sim M_G$ and for $\chi_R \gg \omega_{R,Y}$ (standard $SU(5)$ in the first stage).

This is the origin of the common knowledge that nonsupersymmetric $SO(10)$ settings with the adjoint VEVs driving the gauge symmetry breaking are not phenomenologically viable. In particular, a large hierarchy between the 45_H VEVs, that would set the stage for consistent unification patterns, is excluded.

The key question is: why are the masses of the states in Eqs. (16)–(17) so tightly correlated? Equivalently, why do they depend on a_2 only?

IV. UNDERSTANDING THE SCALAR SPECTRUM

A detailed comprehension of the patterns in the scalar spectrum may be achieved by understanding the correlations between mass textures and the symmetries of the scalar potential. In particular, the appearance of accidental global symmetries in limiting cases may provide the rationale for the dependence of mass eigenvalues from specific couplings. To this end we classify the most interesting cases, providing a counting of the would-be Goldstone bosons (WGB) and pseudo Goldstone bosons (PGB) for each case. A side benefit of this discussion is a consistency check of the explicit form of the mass spectra.

A. 45 only with $a_2 = 0$

Let us first consider the potential generated by 45_H, namely V_Φ in Eq. (3). When $a_2 = 0$, i.e. when only trivial 45_H invariants (built off moduli) are considered, the scalar potential exhibits an enhanced global symmetry: $O(45)$. The spontaneous symmetry breaking (SSB) triggered by the 45_H VEV reduces the global symmetry to $O(44)$. As a consequence, 44 massless states are expected in the scalar spectrum. This is verified explicitly in Appendix C 2 a. Considering the case of the $SO(10)$ gauge symmetry broken to the flipped $SU(5)' \otimes U(1)_{Z'}$, $45 - 25 = 20$ WGB, with the quantum numbers of the coset $SO(10)/SU(5)' \otimes U(1)_{Z'}$ algebra, decouple from the physical spectrum while, $44 - 20 = 24$ PGB remain, whose mass depends on the explicit breaking term a_2.

B. 16 only with $\lambda_2 = 0$

We proceed in analogy with the previous discussion. Taking $\lambda_2 = 0$ in V_χ enhances the global symmetry to $O(32)$. The spontaneous breaking of $O(32)$ to $O(31)$ due to the 16_H VEV leads to 31 massless modes, as it is explicitly seen in Appendix C 2 b. Since the gauge $SO(10)$ symmetry is broken by χ_R to the standard $SU(5)$, $45 - 24 = 21$ WGB, with the quantum numbers of the coset $SO(10)/SU(5)$ algebra, decouple from the physical spectrum, while $31 - 21 = 10$ PGB do remain. Their masses depend on the explicit breaking term λ_2.

C. A trivial 45-16 potential ($a_2 = \lambda_2 = \beta = \tau = 0$)

When only trivial invariants (i.e. moduli) of both 45_H and 16_H are considered, the global symmetry of V_0 in Eq. (3) is $O(45) \otimes O(32)$. This symmetry is spontaneously broken into $O(44) \otimes O(31)$ by the 45_H and 16_H VEVs yielding 44+31=75 GB in the scalar spectrum (see Appendix C 2 d). Since in this case, the gauge $SO(10)$ symmetry is broken to the SM gauge group,

$45 - 12 = 33$ WGB, with the quantum numbers of the coset $SO(10)/SM$ algebra, decouple from the physical spectrum, while $75 - 33 = 42$ PGB remain. Their masses are generally expected to receive contributions from the explicitly breaking terms a_2, λ_2, β and τ.

D. A trivial 45-16 interaction ($\beta = \tau = 0$)

Turning off just the β and τ couplings still allows for independent global rotations of the Φ and χ Higgs fields. The largest global symmetries are those determined by the a_2 and λ_2 terms in V_0, namely $O(10)_{45}$ and $O(10)_{16}$, respectively. Consider the spontaneous breaking to global flipped $SU(5)' \otimes U(1)_{Z'}$ and the standard $SU(5)$ by the 45_H and 16_H VEVs, respectively. This setting gives rise to $20 + 21 = 41$ massless scalar modes. The gauged $SO(10)$ symmetry is broken to the SM group so that 33 WGB decouple from the physical spectrum. Therefore, 41-33=8 PGB remain, whose masses receive contributions from the explicit breaking terms β and τ. All of these features are readily verified by inspection of the scalar mass spectrum in Appendix C 2 e.

E. A tree-level accident

The tree-level masses of the crucial $(1, 3, 0)$ and $(8, 1, 0)$ multiplets belonging to the 45_H depend only on the parameter a_2 but *not* on the other parameters expected (c.f. IV C), namely λ_2, β and τ.

While the λ_2 and τ terms cannot obviously contribute at the tree level to 45_H mass terms, one would generally expect a contribution from the β term, proportional to χ_R^2. Using the parametrization $\Phi = \sigma_{ij}\phi_{ij}/4$, where the σ_{ij} $(i, j \in \{1, .., 10\}, i \neq j)$ matrices represent the $SO(10)$ algebra on the 16-dimensional spinor basis (c.f. Appendix A), one obtains a 45_H mass term of the form

$$\frac{\beta}{16}\chi_R^2 \ (\sigma_{ij})_{16\beta}(\sigma_{kl})_{\beta 16} \ \phi_{ij}\phi_{kl} \,. \qquad (19)$$

The projection of the ϕ_{ij} fields onto the $(1, 3, 0)$ and $(8, 1, 0)$ components lead, as we know, to vanishing contributions.

This result can actually be understood on general grounds by observing that the scalar interaction in Eq. (19) has the same structure as the gauge boson mass from the covariant-derivative interaction with the 16_H, c.f. Eq. (C4). As a consequence, no tree-level mass contribution from the β coupling can be generated for the 45_H scalars carrying the quantum numbers of the standard $SU(5)$ algebra.

This behavior can be again verified by inspecting the relevant scalar spectra in Appendix C 2.

The above considerations provide a clear rationale for the accidental tree level constraint on ω_Y/ω_R, that holds independently on the size of χ_R.
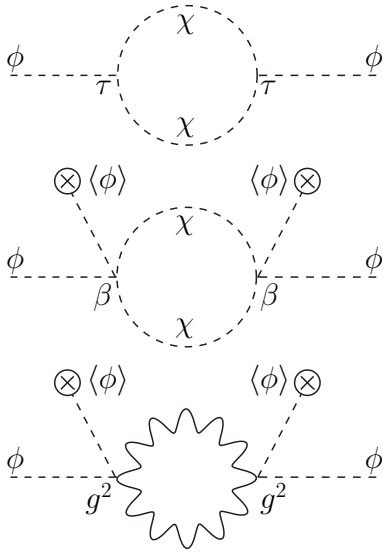
FIG. 1. Typical one-loop diagrams that induce for $\langle\chi\rangle = 0$, $O(\tau/4\pi, \beta\langle\phi\rangle/4\pi, g^2\langle\phi\rangle/4\pi)$ renormalization to the mass of 45_H fields at the unification scale. They are relevant for the PGB states, whose tree level mass is proportional to a_2.

On the other hand, we should expect the β and τ interactions to contribute $O(M_G/4\pi)$ terms to the masses of $(1,3,0)$ and $(8,1,0)$ at the quantum level.

Similar contributions should also arise from the gauge interactions, that break explicitly the independent global transformations on the 45_H and 16_H discussed in the previous subsections.

The typical one-loop self energies, proportional to the 45_H VEVs, are diagrammatically depicted in Fig. 1. While the exchange of 16_H components is crucial, the χ_R is not needed to obtain the large mass shifts. In the phenomenologically allowed unification patterns it gives actually negligible contributions.

It is interesting to notice that the τ-induced mass corrections do not depend on the gauge symmetry breaking, yielding an $SO(10)$ symmetric contribution to all scalars in 45_H.

One is thus lead to the conclusion that any result based on the particular shape of the tree-level 45_H vacuum is drastically affected at the quantum level. Let us emphasize that although one may in principle avoid the τ-term by means of e.g. an extra Z_2 symmetry, no symmetry can forbid the β-term and the gauge loop contributions.

In case one resorts to 126_H, in place of 16_H, for the purpose of $B-L$ breaking, the more complex tensor structure of the class of $126_H^\dagger 45_H^2 126_H$ quartic invariants in the scalar potential may admit tree-level contributions to the states $(1,3,0)$ and $(8,1,0)$ proportional to $\langle 126_H\rangle$. On the other hand, as mentioned above, whenever $\langle 126_H\rangle$ is small on the unification scale, the same considerations apply, as for the 16_H case.

F. The $\chi_R = 0$ limit

From the previous discussion it is clear that the answer to the question whether the non-$SU(5)$ vacua are allowed at the quantum level is independent on the specific value of the $B-L$ breaking VEV ($\chi_R \ll M_G$ in potentially realistic cases).

In order to simplify the study of the scalar potential beyond the classical level it is therefore convenient (and sufficient) to consider the $\chi_R = 0$ limit.

When $\chi_R = 0$ the mass matrices of the 45_H and 16_H sectors are not coupled. The stationary equations in Eqs. (11)–(12) lead to the four solutions

- $\omega = \omega_R = \omega_Y$ ($5\,1_Z$)

- $\omega = \omega_R = -\omega_Y$ ($5'\,1_{Z'}$)

- $\omega_R = 0$ and $\omega_Y \neq 0$ ($3_c\,2_L\,2_R\,1_X$)

- $\omega_R \neq 0$ and $\omega_Y = 0$ ($4_C\,2_L\,1_R$)

In what follows, we will focus our discussion on the last three cases only.

It is worth noting that the tree level spectrum in the $\chi_R = 0$ limit is not directly obtained from the general formulae given in Appendix C 2 c, since Eq. (13) is trivially satisfied for $\chi_R = 0$. The corresponding scalar mass spectra are derived and discussed in Appendix C 2 f. Yet again, it is apparent that the non $SU(5)$ vacuum configurations exhibit unavoidable tachyonic states in the scalar spectrum.

V. THE QUANTUM VACUUM

A. The one-loop effective potential

We shall compute the relevant one-loop corrections to the tree level results by means of the one-loop effective potential (effective action at zero momentum) [26]. We can formally write

$$V_{\text{eff}} = V_0 + \Delta V_s + \Delta V_f + \Delta V_g\,, \qquad (20)$$

where V_0 is the tree level potential and $\Delta V_{s,f,g}$ denote the quantum contributions induced by scalars, fermions and gauge bosons respectively. In dimensional regularization with the modified minimal subtraction ($\overline{MS}$) and in the Landau gauge, they are given by

$$\Delta V_s(\phi, \chi, \mu) = \frac{\eta}{64\pi^2}\text{Tr}\left[W^4(\phi,\chi)\left(\log\frac{W^2(\phi,\chi)}{\mu^2} - \frac{3}{2}\right)\right], \quad (21)$$

$$\Delta V_f(\phi, \chi, \mu) = -\frac{\kappa}{64\pi^2}\text{Tr}\left[M^4(\phi,\chi)\left(\log\frac{M^2(\phi,\chi)}{\mu^2} - \frac{3}{2}\right)\right], \quad (22)$$

$$\Delta V_g(\phi, \chi, \mu) =$$
$$\frac{3}{64\pi^2} \mathrm{Tr} \left[\mathcal{M}^4(\phi, \chi) \left(\log \frac{\mathcal{M}^2(\phi, \chi)}{\mu^2} - \frac{5}{6} \right) \right] , \quad (23)$$

with $\eta = 1(2)$ for real (complex) scalars and $\kappa = 2(4)$ for Weyl (Dirac) fermions. W, M and $\mathcal{M}$ are the functional scalar, fermion and gauge boson mass matrices respectively, as obtained from the tree level potential.

In the case at hand, we may write the functional scalar mass matrix, $W^2(\phi, \chi)$ as a 77-dimensional hermitian matrix, with a lagrangian term

$$\frac{1}{2} \psi^\dagger W^2 \psi , \quad (24)$$

defined on the vector basis $\psi = (\phi, \chi, \chi^*)$. More explicitly, W^2 takes the block form

$$W^2(\phi, \chi) = \begin{pmatrix} V_{\phi\phi} & V_{\phi\chi} & V_{\phi\chi^*} \\ V_{\chi^*\phi} & V_{\chi^*\chi} & V_{\chi^*\chi^*} \\ V_{\chi\phi} & V_{\chi\chi} & V_{\chi\chi^*} \end{pmatrix} , \quad (25)$$

where the subscripts denote the derivatives of the scalar potential with respect to the set of fields ϕ, χ and χ^*. In the one-loop part of the effective potential $V \equiv V_0$.

We neglect the fermionic component of the effective potential since there are no fermions at the GUT scale (we assume that the right-handed (RH) neutrino mass is substantially lower than the unification scale).

The functional gauge boson mass matrix, $\mathcal{M}^2(\phi, \chi)$ is given in Appendix C, Eqs. (C3)–(C4).

B. The one-loop stationary equations

The first derivative of the one-loop part of the effective potential, with respect to the scalar field component ψ_a, reads

$$\frac{\partial \Delta V_s}{\partial \psi_a} = \frac{1}{64\pi^2} \mathrm{Tr} \left[\{ W_{\psi_a}^2, W^2 \} \right.$$
$$\left. \times \left(\log \frac{W^2}{\mu^2} - \frac{3}{2} \right) + W^2 W_{\psi_a}^2 \right] \quad (26)$$

where the symbol $W_{\psi_a}^2$ stands for the partial derivative of W^2 with respect to ψ_a. Analogous formulae hold for $\partial \Delta V_{f,g}/\partial \psi_a$. The trace properties ensure that Eq. (26) holds independently on whether W^2 does commute with its first derivatives or not.

The calculation of the loop corrected stationary equations due to gauge bosons and scalar exchange is straightforward (for $\chi_R = 0$ the 45_H and 16_H blocks decouple in Eq. (25)). On the other hand, the corrected equations are quite cumbersome and we do not explicitly report them here. It is enough to say that the quantum analogue of Eq. (12) admits analytically the same solutions as we had at the tree level. Namely, these are $\omega_R = \omega_Y$, $\omega_R = -\omega_Y$, $\omega_R = 0$ and $\omega_Y = 0$, corresponding respectively to the standard $5\,1_Z$, flipped $5'\,1_{Z'}$, $3_c\,2_L\,2_R\,1_X$ and $4_C\,2_L\,1_R$ preserved subalgebras.

C. The one-loop scalar mass

In order to calculate the second derivatives of the one-loop contributions to V_{eff} it is in general necessary to take into account the commutation properties of W^2 with its derivatives that enter as a series of nested commutators. The general expression can be written as

$$\frac{\partial^2 \Delta V_s}{\partial \psi_a \partial \psi_b} = \frac{1}{64\pi^2} \mathrm{Tr} \left[W_{\psi_a}^2 W_{\psi_b}^2 + W^2 W_{\psi_a \psi_b}^2 \right.$$
$$+ \left[\{ W_{\psi_a \psi_b}^2, W^2 \} + \{ W_{\psi_a}^2, W_{\psi_b}^2 \} \right] \left(\log \frac{W^2}{\mu^2} - \frac{3}{2} \right)$$
$$+ \sum_{m=1}^\infty (-1)^{m+1} \frac{1}{m} \sum_{k=1}^m \binom{m}{k} \{ W^2, W_{\psi_a}^2 \}$$
$$\left. \times \left[W^2, .. \left[W^2, W_{\psi_b}^2 \right] .. \right] \left(W^2 - 1 \right)^{m-k} \right] \quad (27)$$

where the commutators in the last line are taken $k - 1$ times. Let us also remark that, although not apparent, the RHS of Eq. (27) can be shown to be symmetric under $a \leftrightarrow b$, as it should be. In specific cases (for instance when the nested commutators vanish or they can be rewritten as powers of a certain matrix commuting with W) the functional mass evaluated on the vacuum may take a closed form.

1. Running and pole mass

The effective potential is a functional computed at zero external momenta. Whereas the stationary equations allow for the localization of the new minimum (being the VEVs translationally invariant), the mass shifts obtained from Eq. (27) define the running masses $\overline{m}_{ab}^2$

$$\overline{m}_{ab}^2 \equiv \left. \frac{\partial^2 V_{\mathrm{eff}}(\phi)}{\partial \psi_a \partial \psi_b} \right|_{\langle \psi \rangle} = m_{ab}^2 + \Sigma_{ab}(0) \quad (28)$$

where m_{ab}^2 are the renormalized masses and $\Sigma_{ab}(p^2)$ are the $\overline{MS}$ renormalized self-energies. The physical (pole) masses M_a^2 are then obtained as a solution to the equation

$$\det \left[p^2 \delta_{ab} - \left(\overline{m}_{ab}^2 + \Delta \Sigma_{ab}(p^2) \right) \right] = 0 \quad (29)$$

where

$$\Delta \Sigma_{ab}(p^2) = \Sigma_{ab}(p^2) - \Sigma_{ab}(0) \quad (30)$$

For a given eigenvalue

$$M_a^2 = \overline{m}_a^2 + \Delta \Sigma_a(M_a^2) \quad (31)$$

gives the physical mass. The gauge and scheme dependence in Eq. (28) is canceled by the relevant contributions from Eq. (30). In particular, infrared divergent terms in Eq. (28) related to the presence of massless WGB in the Landau gauge cancel in Eq. (31).

Of particular relevance is the case when M_a is substantially smaller than the (GUT-scale) mass of the particles that contribute to $\Sigma(0)$. At $\mu = M_G$, in the $M_a^2 \ll M_G^2$ limit, one has

$$\Delta\Sigma_a(M_a^2) = O(M_a^4/M_G^2) . \qquad (32)$$

In this case the running mass computed from Eq. (28) contains the leading gauge independent corrections. As a matter of fact, in order to study the vacua of the potential in Eq. (20), we need to compute the zero momentum mass corrections just to those states that are tachyonic at the tree level and whose corrected mass turns out to be of the order of $M_G/4\pi$.

We may safely neglect the one loop corrections for all other states with masses of order M_G. It is remarkable, as we shall see, that for $\chi_R = 0$ the relevant corrections to the masses of the critical PGB states can be obtained from Eq. (27) with vanishing commutators.

D. One-loop PGB masses

The stringent tree-level constraint on the ratio ω_Y/ω_R, coming from the positivity of the $(1, 3, 0)$ and $(8, 1, 0)$ masses, follows from the fact that some scalar masses depend only on the parameter a_2. On the other hand, the discussion on the would-be global symmetries of the scalar potential shows that in general their mass should depend on other terms in the scalar potential, in particular τ and β.

A set of typical one-loop diagrams contributing $O(\langle\phi\rangle/4\pi)$ renormalization to the masses of 45_H states is depicted in Fig. 1. As we already pointed out the 16_H VEV does not play any role in the leading GUT scale corrections (just the interaction between 45_H and 16_H, or with the massive gauge bosons is needed). Therefore we henceforth work in the strict $\chi_R = 0$ limit, that simplifies substantially the calculation. In this limit the scalar mass matrix in Eq. (25) is block diagonal (c.f. Appendix C 2 f) and the leading corrections from the one-loop effective potential are encoded in the $V_{\chi^*\chi}$ sector.

More precisely, we are interested in the corrections to those 45_H scalar states whose tree level mass depends only on a_2 and have the quantum numbers of the preserved non-abelian algebra (see Sect. IV A and Appendix C 2 f). It turns out that focusing to this set of PGB states the functional mass matrix W^2 and its first derivative do commute for $\chi_R = 0$ and Eq. (27) simplifies accordingly. This allows us to compute the relevant mass corrections in a closed form.

The calculation of the EP running mass from Eq. (27) leads for the states $(1, 3, 0)$ and $(8, 1, 0)$ at $\mu = M_G$ to the mass shifts

$$\Delta M^2(1, 3, 0) = \frac{1}{4\pi^2} \left[\tau^2 + \beta^2(2\omega_R^2 - \omega_R\omega_Y + 2\omega_Y^2) + g^4 \left(16\omega_R^2 + \omega_Y\omega_R + 19\omega_Y^2\right) \right] , \qquad (33)$$

$$\Delta M^2(8, 1, 0) = \frac{1}{4\pi^2} \left[\tau^2 + \beta^2(\omega_R^2 - \omega_R\omega_Y + 3\omega_Y^2) + g^4 \left(13\omega_R^2 + \omega_Y\omega_R + 22\omega_Y^2\right) \right] , \qquad (34)$$

where the sub-leading (and gauge dependent) logarithmic terms are not explicitly reported. For the vacuum configurations of interest we find the results reported in Appendix D. In particular, we obtain

- $\omega = \omega_R = -\omega_Y$ $(5' 1_{Z'})$:

$$M^2(24, 0) = -4a_2\omega^2 + \frac{\tau^2 + (5\beta^2 + 34g^4)\omega^2}{4\pi^2} , \qquad (35)$$

- $\omega_R = 0$ and $\omega_Y \neq 0$ $(3_c 2_L 2_R 1_X)$:

$$M^2(1, 3, 1, 0) = M^2(1, 1, 3, 0) = 2a_2\omega_Y^2 + \frac{\tau^2 + (2\beta^2 + 19g^4)\omega_Y^2}{4\pi^2} , \qquad (36)$$

$$M^2(8, 1, 1, 0) = -4a_2\omega_Y^2 + \frac{\tau^2 + (3\beta^2 + 22g^4)\omega_Y^2}{4\pi^2} , \qquad (37)$$

- $\omega_R \neq 0$ and $\omega_Y = 0$ $(4_C 2_L 1_R)$:

$$M^2(1, 3, 0) = -4a_2\omega_R^2 + \frac{\tau^2 + (2\beta^2 + 16g^4)\omega_R^2}{4\pi^2} , \qquad (38)$$

$$M^2(15, 1, 0) = 2a_2\omega_R^2 + \frac{\tau^2 + (\beta^2 + 13g^4)\omega_R^2}{4\pi^2} . \qquad (39)$$

In the effective theory language Eqs. (35)–(39) can be interpreted as the one-loop GUT-scale matching due to the decoupling of the massive $SO(10)/G$ states where G is the preserved gauge group. These are the only relevant one-loop corrections needed in order to discuss the vacuum structure of the model.

It is quite apparent that a consistent scalar mass spectrum can be obtained in all cases, at variance with the tree level result.

In order to fully establish the existence of the non-$SU(5)$ minima at the quantum level one should identify the regions of the parameter space supporting the desired vacuum configurations and estimate their depths. We shall address these issues in the next section.

E. The one-loop vacuum structure

1. Existence of the new vacuum configurations

The existence of the different minima of the one-loop effective potential is related to the values of the parameters a_2, β, τ and g at the scale $\mu = M_G$. For the flipped $5'1_{Z'}$ case it is sufficient, as one expects, to assume the tree level condition $a_2 < 0$. On the other hand, from Eqs. (36)–(39) we obtain

- $\omega_R = 0$ and $\omega_Y \neq 0$ $(3_c\,2_L\,2_R\,1_X)$:

$$-8\pi^2 a_2 < \frac{\tau^2}{\omega_Y^2} + 2\beta^2 + 19g^4\,, \qquad (40)$$

- $\omega_R \neq 0$ and $\omega_Y = 0$ $(4_C\,2_L\,1_R)$:

$$-8\pi^2 a_2 < \frac{\tau^2}{\omega_R^2} + \beta^2 + 13g^4\,. \qquad (41)$$

Considering for naturalness $\tau \sim \omega_{Y,R}$, Eqs. (40)–(41) imply $|a_2| < 10^{-2}$. This constraint remains within the natural perturbative range for dimensionless couplings. While all PGB states whose mass is proportional to $-a_2$ receive large positive loop corrections, quantum corrections are numerically irrelevant for all of the states with GUT scale mass. On the same grounds we may safely neglect the multiplicative a_2 loop corrections induced by the 45_H states on the PGB masses.

2. Absolute minimum

It remains to show that the non $SU(5)$ solutions may actually be absolute minima of the potential. To this end it is necessary to consider the one-loop corrected stationary equations and calculate the vacuum energies in the relevant cases. Studying the shape of the one-loop effective potential is a numerical task. On the other hand, in the approximation of neglecting at the GUT scale the logarithmic corrections, we may reach non-detailed but definite conclusions. For the three relevant vacuum configurations we obtain:

- $\omega = \omega_R = -\omega_Y$ $(5'1_{Z'})$

$$V(\omega, \chi_R = 0) = -\frac{3\nu^4}{16\pi^2} + \left(\frac{5\alpha\nu^2}{\pi^2} + \frac{5\beta\nu^2}{16\pi^2} - \frac{5\tau^2}{16\pi^2}\right)\omega^2$$
$$+ \left(-100a_1 - \frac{65a_2}{4} + \frac{600a_1^2}{\pi^2} - \frac{45a_1a_2}{\pi^2} - \frac{645a_2^2}{32\pi^2} + \frac{100\alpha^2}{\pi^2} + \frac{25\alpha\beta}{2\pi^2} + \frac{65\beta^2}{64\pi^2} - \frac{5g^4}{2\pi^2}\right)\omega^4\,, \qquad (42)$$

- $\omega_R = 0$ and $\omega_Y \neq 0$ $(3_c\,2_L\,2_R\,1_X)$

$$V(\omega_Y, \chi_R = 0) = -\frac{3\nu^4}{16\pi^2} + \left(\frac{3\alpha\nu^2}{\pi^2} + \frac{3\beta\nu^2}{16\pi^2} - \frac{3\tau^2}{16\pi^2}\right)\omega_Y^2$$
$$+ \left(-36a_1 - \frac{21a_2}{4} + \frac{216a_1^2}{\pi^2} + \frac{33a_1a_2}{\pi^2} + \frac{45a_2^2}{32\pi^2} + \frac{36\alpha^2}{\pi^2} + \frac{9\alpha\beta}{2\pi^2} + \frac{21\beta^2}{64\pi^2} - \frac{15g^4}{16\pi^2}\right)\omega_Y^4\,, \qquad (43)$$

- $\omega_R \neq 0$ and $\omega_Y = 0$ $(4_C\,2_L\,1_R)$

$$V(\omega_R, \chi_R = 0) = -\frac{3\nu^4}{16\pi^2} + \left(\frac{2\alpha\nu^2}{\pi^2} + \frac{\beta\nu^2}{8\pi^2} - \frac{\tau^2}{8\pi^2}\right)\omega_R^2$$
$$+ \left(-16a_1 - 2a_2 + \frac{96a_1^2}{\pi^2} + \frac{42a_1a_2}{\pi^2} + \frac{147a_2^2}{32\pi^2} + \frac{16\alpha^2}{\pi^2} + \frac{2\alpha\beta}{\pi^2} + \frac{\beta^2}{8\pi^2} - \frac{7g^4}{16\pi^2}\right)\omega_R^4\,. \qquad (44)$$

A simple numerical analysis reveals that for natural values of the dimensionless couplings and GUT mass parameters any of the qualitatively different vacuum configurations may be a global minimum of the one-loop effective potential in a large domain of the parameter space.

This concludes the proof of existence of all of the group-theoretically allowed vacua. Nonsupersymmetric $SO(10)$ models broken at M_G by the 45_H SM preserving VEVs, do exhibit at the quantum level the full spectrum of intermediate symmetries. This is crucially relevant for those chains that, allowed by gauge unification, are accidentally excluded by the tree level potential.

F. The extended survival hypothesis

In a realistic $SO(10)$ unification setup, throughout all the stages of the symmetry breaking one usually assumes that the scalar spectrum obeys the so called extended survival hypothesis (ESH) that reads [27]: "at every stage of the symmetry breaking chain only those scalars are present that develop a VEV at the current or the subsequent levels of the spontaneous symmetry breaking".

The ESH is equivalent to performing the minimal number of fine-tunings imposed onto the scalar potential so that all the symmetry breaking steps are obtained at the desired scales. On the technical side one must identify all the Higgs multiplets needed by the breaking pattern and tune their mass according to the gauge symmetry down to the scale of their VEVs. The effects of the presence of these states at intermediate scales has been considered in our recent analysis of non supersymmeteric $SO(10)$ unification patterns [15], up to one exception that we shall now shortly comment upon.

The relevant patterns preserve in the first stage the group $3_c\, 2_L\, 2_R\, 1_X$ (for $\omega_R = 0$) and $4_C\, 2_L\, 1_R$ (for $\omega_Y = 0$). The breaking to the SM gauge group $3_c\, 2_L\, 1_Y$ is achieved by means of the VEV χ_R, constrained to stay at an intermediate scale by gauge unification. Minimally, one must therefore maintain at this scale either of the 16_H multiplets $(1, 1, 2, \frac{1}{2})$ and $(\overline{4}, 1, -\frac{1}{2})$, in the $3_c\, 2_L\, 2_R\, 1_X$ and $4_C\, 2_L\, 1_R$ cases respectively.

As one can see from the scalar spectrum given in Appendix C 2 f, in the $3_c\, 2_L\, 2_R\, 1_X$ vacuum, the scalars $(1, 2, 1, \frac{1}{2})$ and $(1, 1, 2, -\frac{1}{2})$ receive a mass contribution that is linear in the D-odd VEV ω_Y and that breaks their degeneracy. Thus, just the RH doublet $(1, 1, 2, \frac{1}{2})$, which contains the field acquiring the VEV χ_R, may be minimally fine-tuned at that mass scale.

Turning on $\omega_Y \neq 0$ or $\omega_R \neq 0$ at the χ_R scale leads to a non-minimal set of Higgs states at the intermediate scale [15], namely the 45_H multiplets $(1, 1, 3, 0)$ and $(15, 1, 0)$ in the $3_c\, 2_L\, 2_R\, 1_X$ and in the $4_C\, 2_L\, 1_R$ setting respectively (these are the accidentally tachyonic states at the tree level). Inspection of the one-loop mass spectra shows that the needed minimal fine-tuning can be indeed performed.

It is worth noticing that although in the $3_c\, 2_L\, 2_R\, 1_X$ stage D-parity is broken by ω_Y, the masses of the states $(1, 3, 1, 0)$ and $(1, 1, 3, 0)$, depending quadratically on ω_Y (see Appendix C 2 f), do remain degenerate and are *both* tuned at the scale $\omega_R \sim \chi_R$, where the LR symmetry is broken. The presence of the additional LH triplet $(1, 3, 1, 0)$ at the intermediate scale ω_R has a welcome impact on the gauge coupling running. Compared to the results given in [15] for such a breaking pattern, the intermediate $B - L$ scale is raised by almost one order of magnitude (to about 10^{11} GeV), while the GUT scale is slightly lowered to about 10^{16} GeV. Detailed thresholds effects can be considered once the model dependent scalar spectrum is fully worked out.

A final comment is in order. All of the states exchanged in the relevant mass loop corrections in Sect. V D have natural GUT masses. On the other hand, the ESH requires tuning the masses of some of these states at a much lower scale. In a realistic setting, this involves some of the 16_H submultiplets. The fine tuning apparently generates an infrared divergence problem in the one-loop corrections. However, in analogy to our discussion of the WGB contributions to the effective potential in Sect. V C, the infrared divergent terms appearing in the one-loop zero momentum mass corrections disappear when considering the corrections to the physical pole masses. Thus, they can be safely discarded.

VI. SUMMARY AND OUTLOOK

In this paper, we have scrutinized the longstanding result that the class of the minimal nonsupersymmetric $SO(10)$ unified models, with the GUT symmetry broken by the VEVs of the 45-dimensional adjoint representation, cannot provide a successful gauge unification. This common knowledge was based on the observation that the tree level dynamics of the minimal scalar sector allowed only for "$SU(5)$" breaking patterns. This, in turn, clashes with the intermediate symmetries required by nonsupersymmetric $SO(10)$ unification, that enforce an intermediate threshold well below the GUT scale.

We argued that the old result is an artifact of the tree level Higgs potential and showed that quantum corrections have a dramatic impact. The minimization of the one-loop effective potential in the paradigmatic $\chi_R = 0$ limit shows that the simplest $SO(10)$ model with a $45_H \oplus 16_H$ Higgs sector allows for any of the intermediate symmetry patterns available to the pair of the SM-preserving VEVs in 45_H. In particular, the $SU(4)_C \otimes SU(2)_L \otimes U(1)_R$ and $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ chains are supported. Our result generally applies to any Higgs sector where the vacuum is dominated by the 45_H VEVs.

This observation opens the option of reconsidering the minimal nonsupersymmetric $SO(10)$ model as a reference framework for model building. Extending the Higgs sector to include one 10_H (together with either one 126_H or one 16_H) provides the playground for exploring the possibility of a realistic and predictive GUT, along the lines of the recent efforts in the supersymmetric context.

Appendix A: $SO(10)$ algebra representations

We briefly recall here for convenience the basics of $SO(10)$ algebra representations. For a general introduction see Refs. [28, 29].

1. Tensorial representations

The hermitian and antisymmetric generators of the fundamental representation of $SO(10)$ are given by

$$(\epsilon_{ij})_{ab} = -i(\delta_{a[i}\delta_{bj]}), \tag{A1}$$

where $a, b, i, j = 1, .., 10$ and the square bracket stands for anti-symmetrization. They satisfy the $SO(10)$ commutation relations

$$[\epsilon_{ij}, \epsilon_{kl}] = -i(\delta_{jk}\epsilon_{il} - \delta_{ik}\epsilon_{jl} - \delta_{jl}\epsilon_{ik} + \delta_{il}\epsilon_{jk}), \tag{A2}$$

with normalization

$$\text{Tr}\,\epsilon_{ij}\epsilon_{kl} = 2\,\delta_{i[k}\delta_{jl]} \tag{A3}$$

and Dynkin index 2.

The fundamental (vector) representation ϕ_a ($a = 1, ..., 10$) transforms as

$$\phi_a \to \phi_a - \frac{i}{2}\lambda_{ij}(\epsilon_{ij}\phi)_a, \tag{A4}$$

where λ_{ij} are the infinitesimal parameters of the transformation.

The adjoint representation is then obtained as the antisymmetric part of the 2-index $10_a \otimes 10_b$ tensor ϕ_{ab} ($a, b = 1, .., 10$) and transforms as

$$\phi_{ab} \to \phi_{ab} - \frac{i}{2}\lambda_{ij}\,[\epsilon_{ij}, \phi]_{ab}. \tag{A5}$$

Notice that $[\epsilon_{ij}, \phi]^T = -[\epsilon_{ij}, \phi]$ and $[\epsilon_{ij}, \phi]^\dagger = [\epsilon_{ij}, \phi]$.

2. Spinorial representations

Following the notation of Ref. [21], the $SO(10)$ generators S_{ij} ($i, j = 0, .., 9$) acting on the 32-dimensional spinor Ξ are defined as

$$S_{ij} = \frac{1}{4i}\,[\Gamma_i, \Gamma_j], \quad \{\Gamma_i, \Gamma_j\} = 2\delta_{ij}, \tag{A6}$$

with an explicit representation given by

$$\Gamma_0 = \begin{pmatrix} 0 & I_{16} \\ I_{16} & 0 \end{pmatrix}, \quad \Gamma_p = \begin{pmatrix} 0 & is_p \\ -is_p & 0 \end{pmatrix}, \quad p = 1, ..., 9, \tag{A7}$$

where the s_p matrices are defined as ($k = 1, .., 3$)

$$s_k = \eta_k\rho_3, \quad s_{k+3} = \sigma_k\rho_1, \quad s_{k+6} = \tau_k\rho_2. \tag{A8}$$

The matrices σ_k, τ_k, η_k and ρ_k, are given by the following tensor products of 2×2 matrices

$$\begin{aligned} \sigma_k &= I_2 \otimes I_2 \otimes I_2 \otimes \Sigma_k, \\ \tau_k &= I_2 \otimes I_2 \otimes \Sigma_k \otimes I_2, \\ \eta_k &= I_2 \otimes \Sigma_k \otimes I_2 \otimes I_2, \\ \rho_k &= \Sigma_k \otimes I_2 \otimes I_2 \otimes I_2, \end{aligned} \tag{A9}$$

where Σ_k stand for the ordinary Pauli matrices. Defining

$$s_{pq} = \frac{1}{2i}\,[s_p, s_q] \tag{A10}$$

for $p, q = 1, .., 9$, the algebra (A6) is represented by

$$S_{p0} = \frac{1}{2}\begin{pmatrix} s_p & 0 \\ 0 & -s_p \end{pmatrix}, \quad S_{pq} = \frac{1}{2}\begin{pmatrix} s_{pq} & 0 \\ 0 & s_{pq} \end{pmatrix}. \tag{A11}$$

The Cartan subalgebra is spanned over S_{03}, S_{12}, S_{45}, S_{78} and S_{69}. One can construct a chiral projector Γ_χ, that splits the 32-dimensional spinor Ξ into a pair of irreducible 16-dimensional components:

$$\Gamma_\chi = 2^{-5}S_{03}S_{12}S_{45}S_{78}S_{69} = \begin{pmatrix} -I_{16} & 0 \\ 0 & I_{16} \end{pmatrix}. \tag{A12}$$

It is readily verified that Γ_χ has the following properties: $\Gamma_\chi^2 = I_{32}$, $\{\Gamma_\chi, \Gamma_i\} = 0$ and hence $[\Gamma_\chi, S_{ij}] = 0$. Introducing the chiral projectors $P_\pm = \frac{1}{2}(I_{32} \mp \Gamma_\chi)$, the irreducible chiral spinors are defined as

$$\chi_+ = P_+\Xi \equiv \begin{pmatrix} \chi \\ 0 \end{pmatrix}, \quad \chi_- = P_-\Xi \equiv \begin{pmatrix} 0 \\ \chi^c \end{pmatrix}, \tag{A13}$$

where $\chi^c \equiv C\chi^*$ and C is the $SO(10)$ charge conjugation matrix (see next subsection). Analogously, we can use the chiral projectors to write S_{ij} as

$$S_{ij} = P_+S_{ij}P_+ + P_-S_{ij}P_- \equiv \frac{1}{2}\begin{pmatrix} \sigma_{ij} & 0 \\ 0 & \tilde{\sigma}_{ij} \end{pmatrix}, \tag{A14}$$

where the properties $[P_\pm, S_{ij}] = 0$, $P_\pm^2 = P_\pm$ and $P_+ + P_- = I_{32}$ were used.

Finally, matching Eq. (A14) with Eq. (A11), one identifies the hermitian generators $\sigma_{ij}/2$ and $\tilde{\sigma}_{ij}/2$ acting on the χ and χ^c spinors, respectively, as

$$\sigma_{p0} = s_p, \quad \sigma_{pq} = s_{pq}, \quad \tilde{\sigma}_{p0} = -s_p, \quad \tilde{\sigma}_{pq} = s_{pq}. \tag{A15}$$

From their normalization

$$\tfrac{1}{4}\text{Tr}\,\sigma_{ij}\sigma_{kl} = \tfrac{1}{4}\text{Tr}\,\tilde{\sigma}_{ij}\tilde{\sigma}_{kl} = 4\,\delta_{i[k}\delta_{jl]}, \tag{A16}$$

we recover the Dynkin index 4 of the 16-dimensional spinorial representation.

It is convenient to trace out the σ-matrices in the invariants built off the adjoint representation in the natural

basis $\Phi \equiv \sigma_{ij}\phi_{ij}/4$. From the traces of two and four σ-matrices one obtains

$$\text{Tr}\,\Phi^2 = -2\text{Tr}\,\phi^2\,, \tag{A17}$$

$$\text{Tr}\,\Phi^4 = \tfrac{3}{4}\left(\text{Tr}\,\phi^2\right)^2 - \text{Tr}\,\phi^4\,. \tag{A18}$$

In order to maintain a consistent notation, from now on we shall label the indices of the spinorial generators from 1 to 10, and use the following mapping from the basis of Ref. [21] into the basis of Ref. [7] for both vectors and tensors: $\{0312457869\} \to \{12345678910\}$.

3. The charge conjugation C

According to the notation of the previous subsection, the spinor χ and its complex conjugate χ^* transform as

$$\chi \to \chi - \frac{i}{4}\lambda_{ij}\sigma_{ij}\chi\,, \quad \chi^* \to \chi^* + \frac{i}{4}\lambda_{ij}\sigma_{ij}^T\chi^*\,. \tag{A19}$$

The charge conjugated spinor $\chi^c \equiv C\chi^*$ obeys

$$\chi^c \to \chi^c - \frac{i}{4}\lambda_{ij}\tilde{\sigma}_{ij}\chi^c\,, \tag{A20}$$

and thus C satisfies

$$C^{-1}\tilde{\sigma}_{ij}C = -\sigma_{ij}^T\,. \tag{A21}$$

Taking into account Eq. (A9), a formal solution reads

$$C = \sigma_2\tau_2\eta_2\rho_2\,, \tag{A22}$$

which in our basis yields

$$C = \text{antidiag}(+1,-1,-1,+1,-1,+1,+1,-1,$$
$$-1,+1,+1,-1,+1,-1,-1,+1)\,, \tag{A23}$$

and hence $C = C^* = C^{-1} = C^T = C^\dagger$.

4. The Cartan generators

It is convenient to write the five $SO(10)$ Cartan generators in the $3_c\,2_L\,2_R\,1_X$ basis ($X = (B-L)/2$), where the physical interpretation is obvious. For the spinorial representation we have

$$T_R^{(3)} = \tfrac{1}{4}(\sigma_{12} + \sigma_{34})\,, \quad \widetilde{T}_R^{(3)} = \tfrac{1}{4}(-\sigma_{12} + \sigma_{34})\,,$$
$$T_L^{(3)} = \tfrac{1}{4}(\sigma_{34} - \sigma_{12})\,, \quad \widetilde{T}_L^{(3)} = \tfrac{1}{4}(\sigma_{34} + \sigma_{12})\,,$$
$$T_c^{(3)} = \widetilde{T}_c^{(3)} = \tfrac{1}{4}(\sigma_{56} - \sigma_{78})\,,$$
$$T_c^{(8)} = \widetilde{T}_c^{(8)} = \tfrac{1}{4\sqrt{3}}(\sigma_{56} + \sigma_{78} - 2\sigma_{910})\,,$$
$$T_X = \widetilde{T}_X = -\tfrac{2}{3}(\sigma_{56} + \sigma_{78} + \sigma_{910})\,. \tag{A24}$$

While the T's act on χ, the $\widetilde{T}$'s (characterized by a sign flip in σ_{1i}) act on χ^c. The normalization of the Cartan generators is chosen according to the usual SM convention. A GUT-consistent normalization across all generators is obtained by rescaling T_X (and $\widetilde{T}_X$) by $\sqrt{3/2}$.

In order to obtain the physical generators acting on the fundamental representation it is enough to replace $\sigma_{ij}/2$ in Eq. (A24) by ϵ_{ij}.

With this information at hand, one can identify the spinor components of χ and χ^c

$$\chi = (\nu, u_1, u_2, u_3, l, d_1, d_2, d_3,$$
$$-d_3^c, d_2^c, d_1^c, -l^c, u_3^c, -u_2^c, -u_1^c, \nu^c)\,, \tag{A25}$$

and

$$\chi^c = (\nu^c, u_1^c, u_2^c, u_3^c, l^c, d_1^c, d_2^c, d_3^c,$$
$$-d_3, d_2, d_1, -l, u_3, -u_2, -u_1, \nu)^*\,, \tag{A26}$$

where a self-explanatory SM notation has been naturally extended into the scalar sector. In particular, the relative signs in Eqs. (A25)–(A26) arise from the charge conjugation of the $SO(6) \sim SU(4)_C$ and $SO(4) \sim SU(2)_L \otimes SU(2)_R$ components of χ and χ^c.

The standard and flipped embeddings of $SU(5)$ commute with two different Cartan generators, Z and Z' respectively:

$$Z = -4T_R^{(3)} + 6T_X\,, \quad Z' = 4T_R^{(3)} + 6T_X\,. \tag{A27}$$

Given the relation $\text{Tr}\,(T_R^{(3)})^2 = \tfrac{3}{2}\text{Tr}\,T_X^2$ one obtains

$$\text{Tr}\,(YZ) = 0\,, \quad \text{Tr}\,(YZ') \neq 0\,, \tag{A28}$$

where $Y = T_R^{(3)} + T_X$ is the weak hypercharge generator.

As a consequence, the standard $SU(5)$ contains the SM group, while $SU(5)'$ has a subgroup $SU(3)_c \otimes SU(2)_L \otimes U(1)_{Y'}$, with

$$Y' = -T_R^{(3)} + T_X\,. \tag{A29}$$

In terms of Z' and of Y' the weak hypercharge reads

$$Y = \tfrac{1}{5}(Z' - Y')\,. \tag{A30}$$

Using the explicit form of the Cartan generators in the vector representation one finds

$$Z' \propto \text{diag}(-1,-1,+1,+1,+1) \otimes \Sigma_2\,, \tag{A31}$$
$$Z \propto \text{diag}(+1,+1,+1,+1,+1) \otimes \Sigma_2\,. \tag{A32}$$

The vacuum configurations $\omega_R = -\omega_Y$ and $\omega_R = \omega_Y$ in Eq. (7) are aligned with the Z' and the Z generator respectively, thus preserving $SU(5)' \otimes U(1)_{Z'}$ and $SU(5) \otimes U(1)_Z$, respectively.

Appendix B: Vacuum stability

The boundedness of the scalar potential is needed in order to ensure the global stability of the vacuum. The requirement that the potential is bounded from below sets non trivial constraints on the quartic interactions. We do not provide a fully general analysis for the whole field space, but limit ourselves to the constraints obtained for the given vacuum directions.

1. $(\omega_R,\, \omega_Y,\, \chi_R) \neq 0$

From the quartic part of the scalar potential $V_0^{(4)}$ one obtains

$$4a_1(2\omega_R^2 + 3\omega_Y^2)^2 + \frac{a_2}{4}(8\omega_R^4 + 21\omega_Y^4 + 36\omega_R^2\omega_Y^2)$$

$$+ \frac{\lambda_1}{4}\chi_R^4 + 4\alpha\chi_R^2(2\omega_R^2 + 3\omega_Y^2)$$

$$+ \frac{\beta}{4}\chi_R^2(2\omega_R + 3\omega_Y)^2 - \frac{\tau}{2}\chi_R^2(2\omega_R + 3\omega_Y) > 0 \quad \text{(B1)}$$

Notice that the λ_2 term vanishes along the 16_H vacuum direction.

2. $\omega_R = \omega_Y = 0,\, \chi_R \neq 0$

Along this direction the quartic potential $V_0^{(4)}$ reads

$$V_0^{(4)} = \tfrac{1}{4}\lambda_1\chi_R^4, \quad \text{(B2)}$$

which implies

$$\lambda_1 > 0. \quad \text{(B3)}$$

3. $\omega = \omega_R = -\omega_Y,\, \chi_R = 0$

From now on, we focus on the $\chi_R = 0$ case, c.f. Sect. IV F. On this orbit the quartic part of the scalar potential reads

$$V_0^{(4)} = \tfrac{5}{4}\omega^4(80a_1 + 13a_2). \quad \text{(B4)}$$

Taking into account that the scalar mass spectrum implies $a_2 < 0$, we obtain

$$a_1 > -\tfrac{13}{80}a_2. \quad \text{(B5)}$$

4. $\omega_R = 0,\, \omega_Y \neq 0,\, \chi_R = 0$

At the tree level this VEV configuration does not correspond to a minimum of the potential. It is nevertheless useful to inspect the stability conditions along this direction. Since

$$V_0^{(4)} = \tfrac{3}{4}(48a_1 + 7a_2)\omega_Y^4, \quad \text{(B6)}$$

boundedness is obtained, independently on the sign of a_2, when

$$a_1 > -\tfrac{7}{48}a_2. \quad \text{(B7)}$$

5. $\omega_R \neq 0,\, \omega_Y = 0,\, \chi_R = 0$

In analogy with the previous case we have

$$V_0^{(4)} = 2(8a_1 + a_2)\omega_R^4, \quad \text{(B8)}$$

which implies the constraint

$$a_1 > -\tfrac{1}{8}a_2. \quad \text{(B9)}$$

In the case $a_2 < 0$ the constraint in Eq. (B5) provides the global lower bound on a_1.

Appendix C: Tree level mass spectra

1. Gauge bosons

From the scalar kinetic terms

$$\tfrac{1}{4}\text{Tr}\,(D_\mu\phi)^\dagger(D^\mu\phi), \quad \text{(C1)}$$

and

$$\tfrac{1}{2}(D_\mu\chi)^\dagger(D^\mu\chi) + \tfrac{1}{2}(D_\mu\chi^c)^\dagger(D^\mu\chi^c), \quad \text{(C2)}$$

one may write the field dependent mass matrices for the gauge bosons as

$$\mathcal{M}_A^2(\phi)_{(ij)(kl)} = \frac{g^2}{2}\text{Tr}\,[\epsilon_{(ij)}, \phi][\epsilon_{(kl)}, \phi], \quad \text{(C3)}$$

$$\mathcal{M}_A^2(\chi)_{(ij)(kl)} = \frac{g^2}{4}\chi^\dagger\{\sigma_{(ij)}, \sigma_{(kl)}\}\chi. \quad \text{(C4)}$$

where (ij), (kl) stand for ordered pairs of indices, and ϵ_{ij} $(\sigma_{ij}/2)$ with $i, j = 1, .., 10$ are the generators of the fundamental (spinor) representation (see Appendix A).

Eqs. (C3)–(C4), evaluated on the generic $(\omega_{R,Y} \neq 0,\, \chi_R \neq 0)$ vacuum, yield the following contributions to the tree level gauge boson masses:

a.   Gauge bosons masses from 45

Focusing on Eq. (C1) one obtains

$$
\begin{aligned}
&\mathcal{M}_A^2(1,1,+1) = 4g^2\omega_R^2\,,\\
&\mathcal{M}_A^2(\overline{3},1,-\tfrac{2}{3}) = 4g^2\omega_Y^2\,,\\
&\mathcal{M}_A^2(1,3,0) = 0\,,\\
&\mathcal{M}_A^2(8,1,0) = 0\,,\\
&\mathcal{M}_A^2(3,2,-\tfrac{5}{6}) = g^2\,(\omega_R-\omega_Y)^2\,,\\
&\mathcal{M}_A^2(3,2,+\tfrac{1}{6}) = g^2\,(\omega_R+\omega_Y)^2\,,\\
&\mathcal{M}_A^2(1,1,0) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\,,
\end{aligned}
\tag{C5}
$$

where the SM singlet matrix is defined on the basis $(\psi_{24}^{45},\,\psi_1^{45})$, with the superscript referring to the original $SO(10)$ representation and the subscript to the standard $SU(5)$ embedding (see Table II).

Note that, in the limits of standard $5\,1_Z$ ($\omega_R=\omega_Y$), flipped $5'\,1_{Z'}$ ($\omega_R=-\omega_Y$), $3_c\,2_L\,2_R\,1_X$ ($\omega_R=0$) and $4_C\,2_L\,1_R$ ($\omega_Y=0$) vacua, we have respectively 25, 25, 15 and 19 massless gauge bosons, as expected.

b.   Gauge bosons masses from 16

The contributions from Eq. (C2) read

$$
\begin{aligned}
&\mathcal{M}_A^2(1,1,+1) = g^2\chi_R^2\,,\\
&\mathcal{M}_A^2(\overline{3},1,-\tfrac{2}{3}) = g^2\chi_R^2\,,\\
&\mathcal{M}_A^2(1,3,0) = 0\,,\\
&\mathcal{M}_A^2(8,1,0) = 0\,,\\
&\mathcal{M}_A^2(3,2,-\tfrac{5}{6}) = 0\,,\\
&\mathcal{M}_A^2(3,2,+\tfrac{1}{6}) = g^2\chi_R^2\,,\\
&\mathcal{M}_A^2(1,1,0) = \begin{pmatrix} \tfrac{3}{2} & \sqrt{\tfrac{3}{2}} \\ \sqrt{\tfrac{3}{2}} & 1 \end{pmatrix} g^2\chi_R^2\,,
\end{aligned}
\tag{C6}
$$

where the last matrix is again spanned over $(\psi_{24}^{45},\,\psi_1^{45})$, yielding

$$
\mathrm{Det}\,\mathcal{M}_A^2(1,1,0) = 0\,, \tag{C7}
$$
$$
\mathrm{Tr}\,\mathcal{M}_A^2(1,1,0) = \tfrac{5}{2}g^2\chi_R^2\,. \tag{C8}
$$

The number of vanishing entries corresponds to the dimension of the $SU(5)$ algebra preserved by the 16_H VEV χ_R.

Summing together the 45_H and 16_H contributions, we recognize 12 massless states, that correspond to the SM gauge bosons.

2.   Anatomy of the scalar spectrum

In order to understand the dependence of the scalar masses on the various parameters in the Higgs potential

we detail the scalar mass spectrum in the relevant limits of the scalar couplings, according to the discussion on the accidental global symmetries in Sect. IV.

a.   45 only

Applying the stationary conditions in Eqs. (11)–(12), to the flipped $5'\,1_{Z'}$ vacuum with $\omega=\omega_R=-\omega_Y$, we find

$$
\begin{aligned}
M^2(24,0) &= -4a_2\omega^2\,,\\
M^2(10,-4) &= 0\,,\\
M^2(1,0) &= 2\,(80a_1+13a_2)\,\omega^2\,,
\end{aligned}
\tag{C9}
$$

and, as expected, the spectrum exhibits 20 WGB and 24 PGB whose mass depends on a_2 only. The required positivity of the scalar masses gives the constraints

$$
a_2 < 0 \qquad \text{and} \qquad a_1 > -\tfrac{13}{80}a_2\,, \tag{C10}
$$

where the second equation coincides with the constraint coming from the stability of the scalar potential (see Eq. (B5) in Appendix B).

b.   16 only

When only the 16_H part of the scalar potential is considered the symmetry is spontaneously broken to the standard $SU(5)$ gauge group. Applying the the stationary Eq. (13) we find

$$
\begin{aligned}
M^2(\overline{5}) &= 2\lambda_2\chi_R^2\,,\\
M^2(10) &= 0\,,\\
M^2(1) &= \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tfrac{1}{2}\lambda_1\chi_R^2\,,
\end{aligned}
\tag{C11}
$$

in the $(\psi_1^{16},\,\psi_{1*}^{16})$ basis, that yields

$$
\begin{aligned}
\mathrm{Det}\,M^2(1) &= 0\,,\\
\mathrm{Tr}\,M^2(1) &= \lambda_1\chi_R^2\,,
\end{aligned}
\tag{C12}
$$

and as expected we count 21 WGB and 10 PGB modes whose mass depends on λ_2 only. The required positivity of the scalar masses leads to

$$
\lambda_2 > 0 \qquad \text{and} \qquad \lambda_1 > 0\,, \tag{C13}
$$

where the second equation coincides with the constraint coming from the stability of the scalar potential (see Eq. (B3) in Appendix B).

c.   Mixed 45-16 spectrum ($\chi_R \neq 0$)

In the general case the unbroken symmetry is the SM group. Applying first the two stationary conditions in

Eq. (11) and Eq. (13) we find the spectrum below. The 2×2 matrices are spanned over the (ψ^{45}, ψ^{16}) basis whereas the 4×4 SM singlet matrix is given in the $(\psi^{45}_{24}, \psi^{45}_1, \psi^{16}_1, \psi^{16}_{1*})$ basis.

$$M^2(1,1,+1) = \begin{pmatrix} \beta\chi_R^2 + 2a_2\omega_Y(\omega_R + \omega_Y) & \chi_R(\tau - 3\beta\omega_Y) \\ \chi_R(\tau - 3\beta\omega_Y) & 2\omega_R(\tau - 3\beta\omega_Y) \end{pmatrix},$$

$$M^2(\overline{3},1,-\tfrac{2}{3}) = \begin{pmatrix} \beta\chi_R^2 + 2a_2\omega_R(\omega_R + \omega_Y) & \chi_R(\tau - \beta(2\omega_R + \omega_Y)) \\ \chi_R(\tau - \beta(2\omega_R + \omega_Y)) & 2\omega_Y(\tau - \beta(2\omega_R + \omega_Y)) \end{pmatrix}, \tag{C14}$$

$$M^2(1,3,0) = 2a_2(\omega_Y - \omega_R)(\omega_Y + 2\omega_R),$$
$$M^2(8,1,0) = 2a_2(\omega_R - \omega_Y)(\omega_R + 2\omega_Y), \tag{C15}$$
$$M^2(3,2,-\tfrac{5}{6}) = 0,$$

$$M^2(3,2,+\tfrac{1}{6}) = \begin{pmatrix} \beta\chi_R^2 + 4a_2\omega_R\omega_Y & \chi_R(\tau - \beta(\omega_R + 2\omega_Y)) \\ \chi_R(\tau - \beta(\omega_R + 2\omega_Y)) & (\omega_R + \omega_Y)(\tau - \beta(\omega_R + 2\omega_Y)) \end{pmatrix},$$

$$M^2(1,2,-\tfrac{1}{2}) = (\omega_R + 3\omega_Y)(\tau - \beta\omega_R) + 2\lambda_2\chi_R^2, \tag{C16}$$
$$M^2(\overline{3},1,+\tfrac{1}{3}) = 2(\omega_R + \omega_Y)(\tau - \beta\omega_Y) + 2\lambda_2\chi_R^2.$$

$M^2(1,1,0) =$

$$\begin{pmatrix} \tfrac{1}{2}\left(3\beta\chi_R^2 + 4\left(a_2\omega_R^2 + a_2\omega_Y\omega_R + (48a_1 + 7a_2)\omega_Y^2\right)\right) & \sqrt{6}\left(\tfrac{\beta\chi_R^2}{2} + 2(16a_1 + 3a_2)\omega_R\omega_Y\right) & -\tfrac{1}{2}\sqrt{3}\chi_R(\tau - 2\beta\omega_R - (16\alpha + 3\beta)\omega_Y) & -\tfrac{1}{2}\sqrt{3}\chi_R(\tau - 2\beta\omega_R - (16\alpha + 3\beta)\omega_Y) \\ \sqrt{6}\left(\tfrac{\beta\chi_R^2}{2} + 2(16a_1 + 3a_2)\omega_R\omega_Y\right) & \beta\chi_R^2 + 2\left(4(8a_1 + a_2)\omega_R^2 + a_2\omega_Y\omega_R + a_2\omega_Y^2\right) & \frac{\chi_R(-\tau + 2(8\alpha + \beta)\omega_R + 3\beta\omega_Y)}{\sqrt{2}} & \frac{\chi_R(-\tau + 2(8\alpha + \beta)\omega_R + 3\beta\omega_Y)}{\sqrt{2}} \\ -\tfrac{1}{2}\sqrt{3}\chi_R(\tau - 2\beta\omega_R - (16\alpha + 3\beta)\omega_Y) & \frac{\chi_R(-\tau + 2(8\alpha + \beta)\omega_R + 3\beta\omega_Y)}{\sqrt{2}} & \tfrac{1}{2}\lambda_1\chi_R^2 & \tfrac{1}{2}\lambda_1\chi_R^2 \\ -\tfrac{1}{2}\sqrt{3}\chi_R(\tau - 2\beta\omega_R - (16\alpha + 3\beta)\omega_Y) & \frac{\chi_R(-\tau + 2(8\alpha + \beta)\omega_R + 3\beta\omega_Y)}{\sqrt{2}} & \tfrac{1}{2}\lambda_1\chi_R^2 & \tfrac{1}{2}\lambda_1\chi_R^2 \end{pmatrix}. \tag{C17}$$

By applying the remaining stationary condition in Eq. (12) one obtains

$$\text{Det } M^2(1,1,+1) = 0,$$
$$\text{Tr } M^2(1,1,+1) = \frac{(\chi_R^2 + 4\omega_R^2)(\tau - 3\beta\omega_Y)}{2\omega_R},$$
$$\text{Det } M^2(\overline{3},1,-\tfrac{2}{3}) = 0,$$
$$\text{Tr } M^2(\overline{3},1,-\tfrac{2}{3}) = \frac{(\chi_R^2 + 4\omega_Y^2)(\tau - \beta(2\omega_R + \omega_Y))}{2\omega_Y}, \tag{C18}$$
$$\text{Det } M^2(3,2,+\tfrac{1}{6}) = 0,$$
$$\text{Tr } M^2(3,2,+\tfrac{1}{6}) = \beta\chi_R^2 + 4a_2\omega_R\omega_Y + (\omega_R + \omega_Y)(\tau - \beta(\omega_R + 2\omega_Y)),$$
$$\text{Rank } M^2(1,1,0) = 3,$$
$$\text{Tr } M^2(1,1,0) = 2\left((32a_1 + 5a_2)\omega_R^2 + 8(6a_1 + a_2)\omega_Y^2 + 2a_2\omega_R\omega_Y\right) + \chi_R^2\left(\tfrac{5}{2}\beta + \lambda_1\right).$$

In Eqs. (C14)–(C18) we recognize the 33 WGB with the quantum numbers of the coset $SO(10)/SM$ algebra.

In using the stationary condition in Eq. (12), we paid attention not to divide by $(\omega_R + \omega_Y)$, since the flipped

vacuum $\omega = \omega_R = -\omega_Y$ is an allowed configuration. On the other hand, we can freely put ω_R and ω_Y into the denominators, as the vacua $\omega_R = 0$ and $\omega_Y = 0$ are excluded at the tree level. The coupling a_2 in Eq. (C18) is understood to obey the constraint

$$4a_2(\omega_R + \omega_Y)\omega_R\omega_Y + \beta\chi_R^2(2\omega_R + 3\omega_Y) - \tau\chi_R^2 = 0. \quad \text{(C19)}$$

d. A trivial 45-16 potential ($a_2 = \lambda_2 = \beta = \tau = 0$)

It is interesting to study the global symmetries of the scalar potential when only the moduli of 45_H and 16_H

appear in the scalar potential. In order to correctly count the corresponding PGB, the $(1,1,0)$ mass matrix in the limit of $a_2 = \lambda_2 = \beta = \tau = 0$ needs to be scrutinized. We find in the $(\psi_{24}^{45}, \psi_1^{45}, \psi_1^{16}, \psi_{1*}^{16})$ basis,

$$M^2(1,1,0) = \begin{pmatrix} 96a_1\omega_Y^2 & 32\sqrt{6}a_1\omega_R\omega_Y & 8\sqrt{3}\alpha\chi_R\omega_Y & 8\sqrt{3}\alpha\chi_R\omega_Y \\ 32\sqrt{6}a_1\omega_R\omega_Y & 64a_1\omega_R^2 & 8\sqrt{2}\alpha\chi_R\omega_R & 8\sqrt{2}\alpha\chi_R\omega_R \\ 8\sqrt{3}\alpha\chi_R\omega_Y & 8\sqrt{2}\alpha\chi_R\omega_R & \frac{1}{2}\lambda_1\chi_R^2 & \frac{1}{2}\lambda_1\chi_R^2 \\ 8\sqrt{3}\alpha\chi_R\omega_Y & 8\sqrt{2}\alpha\chi_R\omega_R & \frac{1}{2}\lambda_1\chi_R^2 & \frac{1}{2}\lambda_1\chi_R^2 \end{pmatrix}, \quad \text{(C20)}$$

with the properties

$$\text{Rank } M^2(1,1,0) = 2,$$
$$\text{Tr } M^2(1,1,0) = 64a_1\omega_R^2 + 96a_1\omega_Y^2 + \lambda_1\chi_R^2. \quad \text{(C21)}$$

As expected from the discussion in Sect. IV, Eqs. (C14)–(C20) in the $a_2 = \lambda_2 = \beta = \tau = 0$ limit exhibit 75 massless modes out of which 42 are PGB.

e. A trivial 45-16 interaction ($\beta = \tau = 0$)

In this limit, the interaction part of the potential consists only of the α term, which is the product of 45_H and 16_H moduli. Once again, in order to correctly count the massless modes we specialize the $(1,1,0)$ matrix to the $\beta = \tau = 0$ limit. In the $(\psi_{24}^{45}, \psi_1^{45}, \psi_1^{16}, \psi_{1*}^{16})$ basis, we find

$$M^2(1,1,0) = \\ \begin{pmatrix} 2\left(a_2\omega_R^2 + a_2\omega_Y\omega_R + (48a_1 + 7a_2)\omega_Y^2\right) & 2\sqrt{6}(16a_1 + 3a_2)\omega_R\omega_Y & 8\sqrt{3}\alpha\chi_R\omega_Y & 8\sqrt{3}\alpha\chi_R\omega_Y \\ 2\sqrt{6}(16a_1 + 3a_2)\omega_R\omega_Y & 2\left(4(8a_1 + a_2)\omega_R^2 + a_2\omega_Y\omega_R + a_2\omega_Y^2\right) & 8\sqrt{2}\alpha\chi_R\omega_R & 8\sqrt{2}\alpha\chi_R\omega_R \\ 8\sqrt{3}\alpha\chi_R\omega_Y & 8\sqrt{2}\alpha\chi_R\omega_R & \frac{1}{2}\lambda_1\chi_R^2 & \frac{1}{2}\lambda_1\chi_R^2 \\ 8\sqrt{3}\alpha\chi_R\omega_Y & 8\sqrt{2}\alpha\chi_R\omega_R & \frac{1}{2}\lambda_1\chi_R^2 & \frac{1}{2}\lambda_1\chi_R^2 \end{pmatrix}, \quad \text{(C22)}$$

with the properties

$$\text{Rank } M^2(1,1,0) = 3,$$
$$\text{Tr } M^2(1,1,0) = 2\left((32a_1 + 5a_2)\omega_R^2 + 8(6a_1 + a_2)\omega_Y^2 + 2a_2\omega_R\omega_Y\right) + \lambda_1\chi_R^2. \quad \text{(C23)}$$

According to the discussion in Sect. IV, upon inspecting Eqs. (C14)–(C18) in the $\beta = \tau = 0$ limit, one finds 41 massless scalar modes of which 8 are PGB.

f. The 45-16 scalar spectrum for $\chi_R = 0$

The application of the stationary conditions in Eqs. (11)–(12) (for $\chi_R = 0$, Eq. (13) is trivially satisfied) leads to four different spectra according to the four vacua: standard $5\,1_Z$, flipped $5'\,1_{Z'}$, $3_c\,2_L\,2_R\,1_X$ and $4_C\,2_L\,1_R$. We specialize our discussion to the last three cases.

The mass eigenstates are conveniently labeled according to the subalgebras of $SO(10)$ left invariant by each

vacuum. With the help of Tables I–II one can easily recover the decomposition in the SM components. In the limit $\chi_R = 0$ the states 45_H and 16_H do not mix. All of the WGB belong to the 45_H, since for $\chi_R = 0$ the 16_H preserves $SO(10)$.

Consider first the case: $\omega = \omega_R = -\omega_Y$ (which preserves the flipped $5'\,1_{Z'}$ group). For the 45_H components we obtain:

$$\begin{aligned} M^2(24,0) &= -4a_2\omega^2\,, \\ M^2(10,-4) &= 0\,, \\ M^2(1,0) &= 2\,(80a_1 + 13a_2)\,\omega^2\,. \end{aligned} \quad \text{(C24)}$$

Analogously, for the 16_H components we get:

$$\begin{aligned} M^2(10,+1) &= \tfrac{1}{4}\left(\omega^2(80\alpha + \beta) + 2\tau\omega - 2\nu^2\right)\,, \\ M^2(\bar{5},-3) &= \tfrac{1}{4}\left(\omega^2(80\alpha + 9\beta) - 6\tau\omega - 2\nu^2\right)\,, \quad \text{(C25)} \\ M^2(1,+5) &= \tfrac{1}{4}\left(5\omega^2(16\alpha + 5\beta) + 10\tau\omega - 2\nu^2\right)\,. \end{aligned}$$

Since the unbroken group is the flipped $5'\,1_{Z'}$ we recognize, as expected, 45-25=20 WGB. When only trivial 45_H invariants (moduli) are considered the global symmetry of the scalar potential is $O(45)$, broken spontaneously by ω to $O(44)$. This leads to 44 GB in the scalar spectrum. Therefore 44-20=24 PGB are left in the spectrum. On general grounds, their masses should receive contributions from all of the explicitly breaking terms a_2, β and τ. As it is directly seen from the spectrum, only the a_2 term contributes at the tree level to $M(24,0)$. By choosing $a_2 < 0$ one may obtain a consistent minimum of the scalar potential. Quantum corrections are not relevant in this case.

Consider then the case $\omega_R = 0$ and $\omega_Y \neq 0$ which preserves the $3_c\,2_L\,2_R\,1_X$ gauge group. For the 45_H components we obtain:

$$\begin{aligned} M^2(1,3,1,0) &= 2a_2\omega_Y^2\,, \\ M^2(1,1,3,0) &= 2a_2\omega_Y^2\,, \\ M^2(8,1,1,0) &= -4a_2\omega_Y^2\,, \\ M^2(3,2,2,-\tfrac{1}{3}) &= 0\,, \\ M^2(\bar{3},1,1,-\tfrac{2}{3}) &= 0\,, \\ M^2(1,1,1,0) &= 2\,(48a_1 + 7a_2)\,\omega_Y^2\,. \end{aligned} \quad \text{(C26)}$$

Analogously, for the 16_H components we get:

$$\begin{aligned} M^2(3,2,1,+\tfrac{1}{6}) &= \tfrac{1}{4}\left(\omega_Y^2(48\alpha + \beta) - 2\tau\omega_Y - 2\nu^2\right)\,, \\ M^2(\bar{3},1,2,-\tfrac{1}{6}) &= \tfrac{1}{4}\left(\omega_Y^2(48\alpha + \beta) + 2\tau\omega_Y - 2\nu^2\right)\,, \\ M^2(1,2,1,-\tfrac{1}{2}) &= \tfrac{1}{4}\left(\omega_Y^2(48\alpha + 9\beta) + 6\tau\omega_Y - 2\nu^2\right)\,, \\ M^2(1,1,2,+\tfrac{1}{2}) &= \tfrac{1}{4}\left(\omega_Y^2(48\alpha + 9\beta) - 6\tau\omega_Y - 2\nu^2\right)\,. \end{aligned}$$
$$\text{(C27)}$$

Worth of a note is the mass degeneracy of the $(1,3,1,0)$ and $(1,1,3,0)$ multiplets which is due to the fact that for $\omega_R = 0$ D-parity is conserved by even ω_Y powers. On the contrary, in the 16_H components the D-parity is broken by the τ term that is linear in ω_Y.

Since the unbroken group is $3_c\,2_L\,2_R\,1_X$ there are 45-15=30 WGB, as it appears from the explicit pattern of the scalar spectrum. When only trivial invariants (moduli terms) of 45_H are considered the global symmetry of the scalar potential is $O(45)$, broken spontaneously to $O(44)$, thus leading to 44 GB in the scalar spectrum. As a consequence 44-30=14 PGB are left in the spectrum. On general grounds, their masses should receive contributions from all of the explicitly breaking terms a_2, β and τ. As it is directly seen from the spectrum, only the a_2 term contributes at the tree level to the mass of the 14 PGB, leading unavoidably to a tachyonic spectrum. This feature is naturally lifted at the quantum level.

Let us finally consider the case $\omega_R \neq 0$ and $\omega_Y = 0$ (which preserves the $4_C\,2_L\,1_R$ gauge symmetry). For the 45_H components we find:

$$\begin{aligned} M^2(15,1,0) &= 2a_2\omega_R^2\,, \\ M^2(1,3,0) &= -4a_2\omega_R^2\,, \\ M^2(6,2,+\tfrac{1}{2}) &= 0\,, \\ M^2(6,2,-\tfrac{1}{2}) &= 0\,, \\ M^2(1,1,+1) &= 0\,, \\ M^2(1,1,0) &= 8\,(8a_1 + a_2)\,\omega_R^2\,. \end{aligned} \quad \text{(C28)}$$

For the 16_H components we obtain:

$$\begin{aligned} M^2(4,2,0) &= 8\alpha\omega_R^2 - \tfrac{1}{2}\nu^2\,, \\ M^2(\bar{4},1,+\tfrac{1}{2}) &= \omega_R^2(8\alpha + \beta) + \tau\omega_R - \tfrac{1}{2}\nu^2\,, \quad \text{(C29)} \\ M^2(\bar{4},1,-\tfrac{1}{2}) &= \omega_R^2(8\alpha + \beta) - \tau\omega_R - \tfrac{1}{2}\nu^2\,. \end{aligned}$$

The unbroken gauge symmetry in this case corresponds to $4_C\,2_L\,1_R$. Therefore, one can recognize 45-19=26 WGB in the scalar spectrum. When only trivial (moduli) 45_H invariants are considered the global symmetry of the scalar potential is $O(45)$, which is broken spontaneously by ω_R to $O(44)$. This leads globally to 44 massless states in the scalar spectrum. As a consequence, 44-26=18 PGB are left in the 45_H spectrum, that should receive mass contributions from the explicitly breaking terms a_2, β and τ. At the tree level only the a_2 term is present, leading again to a tachyonic spectrum. This is an accidental tree level feature that is naturally lifted at the quantum level.

Appendix D: One-loop mass spectra

We have checked explicitly that the one-loop corrected stationary equation (12) maintains in the $\chi_R = 0$ limit the four tree level solutions, namely, $\omega_R = \omega_Y$, $\omega_R = -\omega_Y$, $\omega_R = 0$ and $\omega_Y = 0$, corresponding respectively to the standard $5\,1_Z$, flipped $5'\,1_{Z'}$, $3_c\,2_L\,2_R\,1_X$ and $4_C\,2_L\,1_R$ vacua.

In what follows we list, for the last three cases, the leading one-loop corrections, arising from the gauge and scalar sectors, to the critical PGB masses. For all other states the loop corrections provide only sub-leading perturbations of the tree-level masses, and as such irrelevant to the present discussion.

1. Gauge contributions to the PGB mass

Before focusing to the three relevant vacuum configurations, it is convenient to write the gauge contribution to the $(1,3,0)$ and $(8,1,0)$ states in the general case.

$$\Delta M^2(1,3,0) = \frac{g^4 \left(16\omega_R^2 + \omega_Y\omega_R + 19\omega_Y^2\right)}{4\pi^2} + \frac{3g^4}{4\pi^2(\omega_R - \omega_Y)}\left[2\left(\omega_R - \omega_Y\right)^3 \log\left(\frac{g^2\left(\omega_R - \omega_Y\right)^2}{\mu^2}\right)\right.$$
$$\left. + \left(4\omega_R - 5\omega_Y\right)\left(\omega_R + \omega_Y\right)^2 \log\left(\frac{g^2\left(\omega_R + \omega_Y\right)^2}{\mu^2}\right) - 4\omega_R^3 \log\left(\frac{4g^2\omega_R^2}{\mu^2}\right) + 8\omega_Y^3 \log\left(\frac{4g^2\omega_Y^2}{\mu^2}\right)\right], \quad (D1)$$

$$\Delta M^2(8,1,0) = \frac{g^4 \left(13\omega_R^2 + \omega_Y\omega_R + 22\omega_Y^2\right)}{4\pi^2} + \frac{3g^4}{8\pi^2(\omega_R - \omega_Y)}\left[\left(\omega_R - \omega_Y\right)^3 \log\left(\frac{g^2\left(\omega_R - \omega_Y\right)^2}{\mu^2}\right)\right.$$
$$\left. + \left(5\omega_R - 7\omega_Y\right)\left(\omega_R + \omega_Y\right)^2 \log\left(\frac{g^2\left(\omega_R + \omega_Y\right)^2}{\mu^2}\right) + 4\omega_Y^2\left(3\omega_R + \omega_Y\right)\log\left(\frac{4g^2\omega_Y^2}{\mu^2}\right) - 8\omega_R^3 \log\left(\frac{4g^2\omega_R^2}{\mu^2}\right)\right]. \quad (D2)$$

One can easily recognize the (tree-level) masses of the gauge bosons in the log's arguments and cofactors (see Appendix C 1 a). Note that only the massive states do contribute to the one-loop correction. (see Sect. V C).

Let's now specialize to the three relevant vacua. First, for the flipped $5'\,1_{Z'}$ case $\omega = \omega_R = -\omega_Y$ one has:

$$\Delta M^2(24,0) = \frac{17g^4\omega^2}{2\pi^2} + \frac{3g^4\omega^2}{2\pi^2}\log\left(\frac{4g^2\omega^2}{\mu^2}\right). \quad (D3)$$

Similarly, for $\omega_R = 0$ and $\omega_Y \neq 0$ $(3_c\,2_L\,2_R\,1_X)$:

$$\Delta M^2(1,3,1,0) = \Delta M^2(1,1,3,0) = \frac{19g^4\omega_Y^2}{4\pi^2} \quad (D4)$$
$$+ \frac{21g^4\omega_Y^2}{4\pi^2}\log\left(\frac{g^2\omega_Y^2}{\mu^2}\right) - \frac{24g^4\omega_Y^2}{4\pi^2}\log\left(\frac{4g^2\omega_Y^2}{\mu^2}\right),$$

$$\Delta M^2(8,1,1,0) = \frac{11g^4\omega_Y^2}{2\pi^2} + \frac{3g^4\omega_Y^2}{2\pi^2}\log\left(\frac{g^2\omega_Y^2}{4\mu^2}\right). \quad (D5)$$

Finally, for $\omega_R \neq 0$ and $\omega_Y = 0$ $(4_C\,2_L\,1_R)$:

$$\Delta M^2(1,3,0) = \frac{4g^4\omega_R^2}{\pi^2} + \frac{3g^4\omega_R^2}{2\pi^2}\log\left(\frac{g^2\omega_R^2}{16\mu^2}\right), \quad (D6)$$

$$\Delta M^2(15,1,0) = \frac{13g^4\omega_R^2}{4\pi^2} + \frac{9g^4\omega_R^2}{4\pi^2}\log\left(\frac{g^2\omega_R^2}{\mu^2}\right)$$
$$- \frac{12g^4\omega_R^2}{4\pi^2}\log\left(\frac{4g^2\omega_R^2}{\mu^2}\right). \quad (D7)$$

2. Scalar contributions to the PGB mass

Since the general formula for the SM vacuum configuration is quite involved, we give directly the corrections to the PGB masses on the three vacua of our interest. We consider first the case $\omega = \omega_R = -\omega_Y$ (flipped $5'\,1_{Z'}$):

$$\Delta M^2(24,0) = \frac{\tau^2 + 5\beta^2\omega^2}{4\pi^2} \quad (D8)$$
$$+ \frac{1}{128\pi^2\omega}\left[(-5\beta\omega - \tau)(5\omega(16\alpha\omega + 5\beta\omega + 2\tau) - 2\nu^2)\log\left(\frac{5\omega^2(16\alpha + 5\beta) + 10\tau\omega - 2\nu^2}{4\mu^2}\right)\right.$$
$$+ \left(\omega\left(3\tau\omega(80\alpha + 3\beta) + \beta\omega^2(27\beta - 400\alpha) - 10\tau^2\right) + \nu^2(10\beta\omega - 6\tau)\right)\log\left(\frac{\omega^2(80\alpha + 9\beta) - 6\tau\omega - 2\nu^2}{4\mu^2}\right)$$
$$\left. + 2\left(\omega\left(\tau(33\beta\omega - 80\alpha\omega) + \beta\omega^2(400\alpha + 17\beta) + 10\tau^2\right) + 2\nu^2(\tau - 5\beta\omega)\right)\log\left(\frac{\omega^2(80\alpha + \beta) + 2\tau\omega - 2\nu^2}{4\mu^2}\right)\right].$$

For $\omega_R = 0$ and $\omega_Y \neq 0$ $(3_c\,2_L\,2_R\,1_X)$, we find:

$$\Delta M^2(1,3,1,0) = \Delta M^2(1,1,3,0) = \frac{\tau^2 + 2\beta^2\omega_Y^2}{4\pi^2} \tag{D9}$$
$$+ \frac{1}{64\pi^2\omega_Y}\left[-(\tau - 3\beta\omega_Y)\left(-3\omega_Y^2(16\alpha + 3\beta) + 6\tau\omega_Y + 2\nu^2\right)\log\left(\frac{\omega_Y^2(48\alpha + 9\beta) - 6\tau\omega_Y - 2\nu^2}{4\mu^2}\right)\right.$$
$$- (\beta\omega_Y + \tau)\left(\omega_Y^2(48\alpha + \beta) + 2\tau\omega_Y - 2\nu^2\right)\log\left(\frac{\omega_Y^2(48\alpha + \beta) + 2\tau\omega_Y - 2\nu^2}{4\mu^2}\right)$$
$$+ \left(3\tau\omega_Y^2(16\alpha - 11\beta) + \beta\omega_Y^3(240\alpha + 17\beta) + 2\omega_Y\left(5\tau^2 - 5\beta\nu^2\right) - 2\nu^2\tau\right)\log\left(\frac{\omega_Y^2(48\alpha + \beta) - 2\tau\omega_Y - 2\nu^2}{4\mu^2}\right)$$
$$\left. + \left(\omega_Y^2(9\beta\tau - 48\alpha\tau) + 3\beta\omega_Y^3(9\beta - 16\alpha) + 2\omega_Y\left(\beta\nu^2 - \tau^2\right) + 2\nu^2\tau\right)\log\left(\frac{\omega_Y^2(48\alpha + 9\beta) + 6\tau\omega_Y - 2\nu^2}{4\mu^2}\right)\right],$$

$$\Delta M^2(8,1,1,0) = \frac{\tau^2 + 3\beta^2\omega_Y^2}{4\pi^2} \tag{D10}$$
$$+ \frac{1}{64\pi^2\omega_Y}\left[-(\tau - 3\beta\omega_Y)\left(-3\omega_Y^2(16\alpha + 3\beta) + 6\tau\omega_Y + 2\nu^2\right)\log\left(\frac{\omega_Y^2(48\alpha + 9\beta) - 6\tau\omega_Y - 2\nu^2}{4\mu^2}\right)\right.$$
$$+ \left(\omega_Y^2(21\beta\tau - 48\alpha\tau) + \beta\omega_Y^3(144\alpha + 11\beta) + \omega_Y\left(6\tau^2 - 6\beta\nu^2\right) + 2\nu^2\tau\right)\log\left(\frac{\omega_Y^2(48\alpha + \beta) + 2\tau\omega_Y - 2\nu^2}{4\mu^2}\right)$$
$$- (3\beta\omega_Y + \tau)\left(\omega_Y^2(48\alpha + 9\beta) + 6\tau\omega_Y - 2\nu^2\right)\log\left(\frac{\omega_Y^2(48\alpha + 9\beta) + 6\tau\omega_Y - 2\nu^2}{4\mu^2}\right)$$
$$\left. + \left(3\tau\omega_Y^2(16\alpha - 7\beta) + \beta\omega_Y^3(144\alpha + 11\beta) + \omega_Y\left(6\tau^2 - 6\beta\nu^2\right) - 2\nu^2\tau\right)\log\left(\frac{\omega_Y^2(48\alpha + \beta) - 2\tau\omega_Y - 2\nu^2}{4\mu^2}\right)\right].$$

Finally, for $\omega_R \neq 0$ and $\omega_Y = 0$ $(4_C\,2_L\,1_R)$, we have:

$$\Delta M^2(1,3,0) = \frac{\tau^2 + 2\beta^2\omega_R^2}{4\pi^2} + \frac{1}{64\pi^2\omega_R}\left[16\omega_R\left(16\alpha\beta\omega_R^2 - \beta\nu^2 + \tau^2\right)\log\left(\frac{8\alpha\omega_R^2 - \frac{\nu^2}{2}}{\mu^2}\right)\right.$$
$$- 4(\tau - 2\beta\omega_R)\left(-2\omega_R^2(8\alpha + \beta) + 2\tau\omega_R + \nu^2\right)\log\left(\frac{\omega_R^2(8\alpha + \beta) - \tau\omega_R - \frac{\nu^2}{2}}{\mu^2}\right)$$
$$\left. - 4(2\beta\omega_R + \tau)\left(2\omega_R^2(8\alpha + \beta) + 2\tau\omega_R - \nu^2\right)\log\left(\frac{\omega_R^2(8\alpha + \beta) + \tau\omega_R - \frac{\nu^2}{2}}{4\mu^2}\right)\right], \tag{D11}$$

$$\Delta M^2(15,1,0) = \frac{\tau^2 + \beta^2\omega_R^2}{4\pi^2}$$
$$+ \frac{1}{64\pi^2\omega_R}\left[8\omega_R\left(16\alpha\beta\omega_R^2 - \beta\nu^2 + \tau^2\right)\log\left(\frac{8\alpha\omega_R^2 - \frac{\nu^2}{2}}{\mu^2}\right)\right.$$
$$- 4\left(2\beta\omega_R^3(8\alpha - \beta) - 16\alpha\tau\omega_R^2 + \omega_R\left(\tau^2 - \beta\nu^2\right) + \nu^2\tau\right)\log\left(\frac{\omega_R^2(8\alpha + \beta) - \tau\omega_R - \frac{\nu^2}{2}}{\mu^2}\right)$$
$$\left. + 4\left(2\beta\omega_R^3(\beta - 8\alpha) - 16\alpha\tau\omega_R^2 + \omega_R\left(\beta\nu^2 - \tau^2\right) + \nu^2\tau\right)\log\left(\frac{\omega_R^2(8\alpha + \beta) + \tau\omega_R - \frac{\nu^2}{2}}{\mu^2}\right)\right]. \tag{D12}$$

Also in these formulae we recognize the (tree level) mass eigenvalues of the 16_H states contributing to the one-loop effective potential (see Appendix C 2 f).

Notice that the singlets with respect to each vacuum, namely $(1,0)$, $(1,1,1,0)$ and $(1,1,0)$, for the flipped $5'\,1_{Z'}$, $3_c\,2_L\,2_R\,1_X$ and $4_C\,2_L\,1_R$ vacua respectively, receive a tree level contribution from both a_1 as well as a_2 (see Appendix C 2 f). The a_1 term leads the tree level mass and radiative corrections can be neglected.

One may verify that in the limit of vanishing VEVs the one-loop masses vanish identically on each of the three vacua, as it should be. This is a non trivial check of the calculation of the scalar induced corrections.

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